Mathematics Olympiad Programme In India

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1 What is a Mathematical Olympiad?

A Mathematical Olympiad is a problem solving competition open to all "Mathletes". The aim of the competition is to test innate problem solving skills. The problems are restricted to those that require minimal background and high ingenuity. Since one of the goals of such a competition is to identify talent at a young age, these olympiads are usually restricted to students not yet admitted to the undergraduate curriculum.

The main international mathematical olympiad is the *International Mathematical Olympiad* (IMO), a competition held annually since 1959. The IMO is essentially an individual competition, although there is an unofficial team ranking. Each country is required to send in a team of *six* members, each yet to join an undergraduate programme and under the age of 20 at the time of the olympiad, held usually in mid July each year. Accompanying each team are two delegates, called the "*Team Leader*" and the "*Team Deputy Leader*", each of whom serve the role of a manager. The Team Leader actually leaves a couple of days in advance of the rest of the team and aside from ensuring the arrangements made are in order, is expected to oversee the academic conduct of his team. The questions for the olympiads and other academic formalities are conducted by a select group of Team Leaders. The IMO is spread over two days, with each member of the Team required to attempt *three* problems in *four and a half* hours on two successive days. Each problem is given a weightage of 7 points, for an individual maximum score of 42 points and a Team total of 252 points. The solutions are initially graded by the Team Leader and Deputy Leader since solutions may be answered in native language. The past few years have seen the participation of around 85 countries, and although there is no official record of ranking among the countries, India has consistently been positioned between ranks 7 and 14 with a medal for each of the six participants. About 8% of all participants receive a gold medal at the IMO, about 16% a silver and about 25% a bronze, so that roughly half of all participants receive some medal.

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2 Organisation of the Mathematics Olympiad Programme in India

The Mathematics Olympiad Programme in India leading to participation in the IMO is currently being organised by the *Homi Bhabha Centre for Science Education* (HBCSE) on behalf of the *National Board of Higher Mathematics* (NBHM) and funded by the *Department of Atomic Energy* (DAE). The current office bearers in-charge of the programme are as follows:

- **National Coordinator:**
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  `Phone: (011) 2656 5050 X 210;`
  `Fax: (011) 2685 6779;`
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- **HBCSE Director:**
  Prof. Arvind Kumar, Centre Director, Homi Bhabha Centre for Science Education, Near Anushaktinagar Bus Depot, V.N. Purav Marg, Mankhurd, Mumbai – 400 088.
  `Phone: (022) 2556 2132, 2556 7711, 2555 5242, 2555 4712;`
  `Fax: (022) 2558 5660, 2556 6803;`
  `e-mail: arvindk@hbsce.tifr.res.in`

- **Member Secretary, NBHM:**
  Dr. D.C. Khandekar, Member Secretary, National Board of Higher Mathematics, Department of Atomic Energy, Anushakti Bhavan, CSM Marg, Mumbai – 400 001.
  `Phone: (022) 2202 2533;`
  `e-mail: nbhm@math.tifr.res.in`

- **Scientists In-Charge:**
  Dr. C.R. Pranesachar, Dr. B.J. Venkatachala, Department of Mathematics, Mathematics Olympiad Cell, Indian Institute of Science, Bangalore – 560 001.
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India has been participating in the IMO since 1989. Ministry of Human Resources Development (MHRD) finances international travel for the 8-member Indian delegation, while NBHM (DAE) finances the entire programme run within the country and expenses other than travel connected with IMO.

3 How can I participate?

There is a three-step procedure in order to represent India at IMO. The minimum qualification in order to do so are as follows:

- must be a citizen of India;

- must not have attended an undergraduate programme anywhere; and
- must be 20 or less at the time of the IMO.

The three-step procedure is as follows:

- **Stage 1: Regional Mathematical Olympiad** (RMO) For the purpose of the conduct of the olympiad programme in our country, India has been divided into 21 regions, some of which are one or a combination of States, while some are cities. In addition to this, CBSE and Navodaya Vidyalaya Samiti (NVS) are also designated as regions, though their students must take the RMO at one of the designated centres in the 21 regions countrywide. Each region has a "*Coordinator*", who is in-charge of the conduct of the RMO in his region. RMO is usually held between September and the first Sunday of December each year at several centres in the 21 different regions in the country. The date of the exam is fixed by the Regional Coordinator. All school students of Classes XI & XII are eligible to appear in RMO. However, the Regional Coordinator reserves the right to admit students of lower classes. The Regional Coordinator also has the freedom to prepare his question paper as RMO or to obtain the paper from the MO Cell in Bangalore. Those opting for the latter option must hold the RMO on the *first Sunday of December*, while other Coordinators may hold it any time prior to that date. RMO as set by the MO Cell is a 3-hour written exam and usually contains $6 - 7$ problems. At present, each region selects 30 students for the next stage, no more than 6 of whom may be from Class XII.

- **Stage 2: Indian National Mathematical Olympiad** (INMO) INMO is held on the *first Sunday of February* each year at one centre of each region. INMO is open only to those selected through RMO from their respective regions or through RMO conducted by CBSE or NVS. INMO is a 4-hour written exam and usually contains 6 questions. It is set by the members of the MO Cell and is common to all candidates. On the basis of INMO, around 30 students from all over the country are selected for the next stage, no more than 6 of whom may be from Class XII.

- **Stage 3: International Mathematical Olympiad Training Camp** (IMOTC) The 30 INMO awardees are invited to a month-long Training Camp held in May-June each year at HBCSE, Mumbai. INMO awardees of previous years who have performed satisfactorily in problems sent by post over the past year are directly invited to the Camp for another round of training as "*Senior Batch*". Training Faculty from all over the country impart problem solving skills alongwith necessary theoretical background to the awardees. The second half of the Camp is also used for selection of the Indian team for IMO. There are *five* exams patterned on the IMO, and on the basis of their performance, the Indian team for IMO is announced on the concluding session of the Camp. All Junior Batch – first time awardees – receive a Certificate of Merit and all Senior Batch receive awards in forms of books and cash to the tune of Rs. 5,000.

All roads to IMO lead through the RMO. To participate in RMO, you must get in touch with your Regional Coordinator. If you are unable to do so, you may contact anyone in the national organisation committee, particularly the scientists at the MO Cell in Bangalore, who will then lead you to your Regional Coordinator.

4 Incentives & Awards for Winners

Participating in a mathematics olympiad is itself an enriching and rewarding experience. If you also happen to do well and qualify for Stage 3 or beyond, there are more rewards in store for you. In addition, Regional Coordinators usually have some awards for their RMO winners even if they have not progressed beyond Stage 1.

- **IMO medallists:** Indian team members who receive a gold, silver or bronze at IMO receive a cash award of Rs. 5,000, Rs. 4,000 and Rs. 3,000 respectively at a formal award ceremony held at the end of the IMOTC the following year.

- **IMO participants:** All Indian team members for IMO automatically qualify for the *Kishore Vaigyanik Protsahan Yojana* (KVPY) Fellowship of Rs. 3,000 per month plus Rs. 6,000 per annum contingency provided they pursue a career in Mathematics or some basic Science.

- **INMO awardees:** INMO awardees choosing *Mathematics* for their undergraduate degree course are provided a scholarship of Rs. 1,000 per month. They are also offered a 4-year programme of training in Mathematics through correspondence and periodic contact with a chosen faculty. The programme is also available to INMO awardees who do not pursue an undergraduate degree in Mathematics but have special interest in the subject. They are offered an annual cash award of Rs. 9,000 subject to satisfactory performance in the programme.

- **INMO awardees:** Each year the faculty from IMOTC selects a maximum of 15 students of Class XII for inclusion in the *Nurture Programme* of NBHM. All six Indian Team members for the IMO are automatically selected as well. Each batch of students is assigned to a faculty of eminent mathematicians, who then devise a syllabus for the students for each academic year and guide them. The students are tested via post, and at the end of the year, called for a "*Contact Programme*" with the faculty for $3-4$ weeks. This time is used in furthering the mathematical education of the student and in assessing the suitability for continuation in the programme.

5 Syllabus for Mathematical Olympiads

There is no prescribed syllabus for mathematical olympiads. Broadly speaking, the topics are taken from pre-college mathematics and the areas covered are Arithmetic of Integers, Geometry, Quadratic equations and expressions, Trigonometry, Coordinate Geometry, Systems of linear equations, Permutations and Combinations, Factorization of polynomials, Inequalities, Elementary Combinatorics, Probability Theory and Number Theory, Finite series and Complex numbers, and Elementary Graph Theory. The major areas from which problems are chosen are Number Theory, Geometry, Algebra and Combinatorics. The syllabus does not include Calculus and Statistics. The problems under each topic are of exceptionally high level in difficulty and sophistication. The difficulty level increases from RMO to INMO to IMO.

6 References

There are a large number of books that may serve as a guide to a Mathlete. Some of these cater directly to the Olympiads at different levels or of different countries, while others are essentially more of a general problem solving nature.

6.1 General Reading

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- N.D. Kazarinoff, *Geometric Inequalities*, New Mathematical Library 4, Random House and The L.W. Singer Co., New York, 1961
- P.P. Korovkin, *Inequalities*, Little Mathematics Library, MIR Publishers, Moscow, 1975
- V. Krishnamurthy, K.N. Ranganathan, B.J. Venkatachala and C.R. Pranesachar, *Challenges and Thrills of Pre-College Mathematics*, Wiley Eastern Ltd., New Delhi, 1991
- I. Niven, H.S. Zuckerman & H.L. Montgomery, *An Introduction to the Theory of Numbers*, Fifth Edition, Wiley Eastern, New Delhi, 2000
- A.W. Tucker, *Applied Combinatorics*, Second Edition, John Wiley & Sons, New York, 1984
- G.N. Yakovlev, *High School Mathematics*, Part II, MIR Publishers, Moscow, 1984

6.2 Olympiad Problem Books

- S.L. Greitzer, *International Mathematical Olympiads 1959-1977*, New Mathematics Library 27, The Mathematical Association of America, New York, 1978; Indian Edition, Mathematical Association of India, New DElhi, 1992
- M.S. Klamkin, *USA Mathematical Olympiads 1972-86*, New Mathematics Library 33, The Mathematical Association of America, New York, 1988; Indian Edition, Mathematical Association of India, New Delhi, 1992
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- V.A. Krechmar, *A Problem Book in Algebra*, MIR Publishers, Moscow, 1974
- M.R. Modak, S.A. Katre & V.V. Acharya (Editors), *An Excursion in Mathematics*, Fourth Edition, Bhaskaracharya Pratishthana, Pune, 2000
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- I.F. Sharygin, *Problems in Plane Geometry*, MIR Publishers, Moscow, 1988
- D.O. Shklyarski, N.N. Chentzov & I.M. Yaglom, *The USSR Olympiad Problem Book: Selected Problems and Theorems in Elementary Mathematics*, MIR Publishers, Moscow, 1979
- H. Steinhaus, *One Hundred Problems in Elementary Mathematics*, Dover Publications, New York, 1964
- A.M. Yaglom & I.M. Yaglom, *Challenging Mathematical Problems with Elementary Solutions*, Vol. 1, Holden Day, San Francisco, London & Amsterdam, 1964

7 Mathematical Olympiad Samplers

Here are some of mathematical olympiad problems:

1. Let a, b, c be psitive real numbers such that $abc = 1$. Prove that
$$\left(a-1+\frac{1}{b}\right)\left(b-1+\frac{1}{c}\right)\left(c-1+\frac{1}{a}\right) \leq 1.$$
(IMO 2000)

2. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ such that
$$f(x-f(y)) = f(f(y)) + xf(y) + f(x) - 1$$
for all real numbers x, y. **(IMO 1999)**

3. Determine all pairs (a, b) of positive integers such that ab^2+b+7 is divides a^2b+a+b. **(IMO 1998)**

4. Angle A is the smallest in the triangle ABC. The points B and C divide the circumcircle of the triangle into two arcs. Let U be an interior point of the arc between B and C which does not contain A. The perpendicular bisectors of AB and AC meet the line AU at V and W, respectively. The lines BV and CW meet at T. Show that $AU = TB + TC$. **(IMO 1998)**

5. Let S denote the set of all 6-tuples (a, b, c, d, e, f) of positive integers such that $a^2 + b^2 + c^2 + d^2 + e^2 = f^2$. Consider the set

$$T = \{abcdef : (a, b, c, d, e, f) \in S\}.$$

   Find the greatest common divisor of all the members of T. **(INMO 2004)**

6. Do there exist 100 lines in the plane, no three of them concurrent, such that they intersect exactly in 2002 points? **(INMO 2002)**

7. Given any nine integers show that it is possible to choose, from among them, four integers a, b, c, d such that $a + b - c - d$ is divisible by 20. Further show that such a selection is not possible if we start with eight integers instead of nine. **(INMO 2001)**

8. Let ABC be a triangle in which $AB = AC$ and $\angle CAB = 90°$. Suppose M and N are points on the hypotenuse BC such that $BM^2 + CN^2 = MN^2$. Prove that $\angle MAN = 45°$. **(RMO 2003)**

9. Find all primes p and q such that $p^2 + 7pq + q^2$ is the square of an integer. **(RMO 2001)**

10. Find all real values of a for which the equation $x^4 - 2ax^2 + x + a^2 - a = 0$ has all its roots real. **(RMO 2000)**