BMath-II-Topology (Back paper)

Instructions: Total time 3 Hours. **All questions are compulsory**. You may use results proved in the class without proof. Use concepts, notations, terminology, results, as covered in the course. If you wish to use a problem from a homework/assignment as a result, supply its solution too.

1. Prove that a connected metric space with at least two points is uncountable.
2. Give an example to show that without the hypothesis that diameter sequence converges to zero, the conclusion of Cantor's intersection theorem is false.
3. Prove that the Cantor set is nowhere dense.
4. Prove that the Klein bottle is Hausdorff.
5. Prove that $\mathrm{SO}(n)/\mathrm{SO}(n-1) \cong S^{n-1}$ for $n \geq 1$.
6. Prove that the group of all topological group automorphisms of the topological group $(\mathbb{R}, +)$ is isomorphic to the group $(\mathbb{R}^{\times}, \cdot)$ of all nonzero real numbers, with usual multiplication.