Indian Statistical Institute, Bangalore
B. Math (III)
First Semester 2019-2020
Mid-Semester Examination : Statistics (III)

Date: 12-09-2019 **Maximum Score 40** **Duration: 3 Hours**

1. Find characteristic function of the random vector $\boldsymbol{Y}$, $(n \times 1)$, whose density function is proportional to $\exp\left[-\frac{1}{2}\boldsymbol{y}'\boldsymbol{A}\boldsymbol{y} - \boldsymbol{y}'\boldsymbol{b} - c\right]$ where $\boldsymbol{A}$ is an $n \times n$ positive definite matrix, $\boldsymbol{b}$ is an $n \times 1$ vector and c is a real number.

[12]

2. If $\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \sim \boldsymbol{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \sigma^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right)$, $\sigma^2 > 0$ and $\rho^2 < 1$, then find a condition on a and b so that that the random variable $W = \frac{(Y_1+aY_2)^2}{(Y_1+bY_2)^2} \times \frac{(b+\rho)^2+1-\rho^2}{(a+\rho)^2+1-\rho^2}$ has F distribution. Under your condition compare the distributions of W and $\frac{1}{W}$.

$[6 + 3 = 9]$

3. Under the simple linear regression model with uncorrelated errors, if β_0 is known then obtain *least squares estimator* for β_1. Find *residual sum of squares.* Hence find an unbiased estimator for σ^2, the error variance.

$[4 + 3 + 5 = 12]$

4. Under the simple linear regression model obtain *likelihood ratio test* (LRT), stating clearly the assumptions that you make, for testing the hypothesis $H_0 : \beta_1 = 0$ vs $H_1 : \beta_1 \neq 0$. Also obtain a 90% confidence interval (CI) for β_1.

$[9 + 3 = 12]$