INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, 2018-19
Statistics - III, Midterm Examination, September 12, 2018
Marks are shown in square brackets. Total Marks: 50
You may use any of the results stated in class by stating them completely

1. Consider a random sample $X_1, \dots, X_n$ from $N(\mu, \sigma^2)$. Let $\bar{X}$ be the sample mean, and define $Y_1 = \bar{X}$, $Y_2 = X_1 - Y_1$ and $Y_3 = X_1 + Y_1$. Find the joint distribution of (Y_1, Y_2, Y_3). [10]

2. Let $Z \sim N(0, 1)$ independent of U which uniformly distributed on the interval $(-1, 1)$. Define $Y = -Z$ if $U \leq 0$ and $Y = Z$ if $U > 0$.
(a) Find the probability distribution of Y.
(b) Show that the joint distribution of Z and Y is not bivariate normal. [10]

3. Suppose $\mathbf{X} \sim N_p(\mathbf{0}, \mathbf{\Sigma})$ where $\text{Rank}(\Sigma) = r \leq p$ and let B and C be any symmetric matrices.
(a) Show that $\mathbf{X}'B\mathbf{X}$ and $\mathbf{X}'C\mathbf{X}$ are independent χ^2 random variables if and only if

$$\Sigma B \Sigma B \Sigma = \Sigma B \Sigma, \ \Sigma C \Sigma C \Sigma = \Sigma C \Sigma, \ \Sigma B \Sigma C \Sigma = \mathbf{0}.$$

(b) Find the degrees of freedom of these χ^2 distributions. [14]

4. Consider the model $\mathbf{Y} = X\beta + \epsilon$, where $X_{n \times p}$ has $\mathbf{1}$ as its first column (i.e., $\mathbf{X}_0$) and has rank $r \leq p$. Assume $\epsilon \sim N_n(\mathbf{0}, \sigma^2 I_n)$ and let β_0 denote the first element of β.
(a) Does the BLUE of β_0 always exist? Justify.
(b) Find the BLUE of β_0 if it exists. What is its probability distribution?
(c) If the BLUE of β_0 exists, what is its joint distribution with RSS? Give a $100(1-\alpha)\%$ confidence interval for β_0. [16]