INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, First Semester, 2006-07
Statistics - III, Midterm Examination, September 22, 2006

(5) 1. If $X_1, \dots, X_n$ are independently and identically distributed with mean θ and variance σ^2, find the expected value of $Q = \sum_{j=1}^{n-1} j(X_j - X_{j+1})^2$.

(7) 2. Let $\mathbf{Y} \sim N_n(\mathbf{0}, \sigma^2 I_n)$ and $\mathbf{a}$ be a non-zero vector. Find the conditional distribution of $\mathbf{Y}'\mathbf{Y}$ given $\mathbf{a}'\mathbf{Y} = 0$.

(13) 3. Consider the model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\epsilon \sim N_n(\mathbf{0}, \sigma^2 I_n)$.
(a) If $\hat{\beta}$ is the least squares estimator of β, show that $(\hat{\beta} - \beta)'\mathbf{X}'\mathbf{X}(\hat{\beta} - \beta)$ is distributed independently of the residual sum of squares.
(b) Consider the case when there is only one regressor, X_1. When do we have independence of $\hat{\beta}_0$ and $\hat{\beta}_1$?
(c) Find the maximum likelihood estimator of σ^2. Is it unbiased?

(5) 4. Let B^- be a generalized inverse of a symmetric matrix B and assume B^- is also symmetric. Show that if $P = BB^-$, then rank of B is the same as trace of P.

(10) 5. Consider the following model:
$y_1 = \theta + \gamma + \epsilon_1$
$y_2 = \theta + \phi + \epsilon_2$
$y_3 = 2\theta + \phi + \gamma + \epsilon_3$
$y_4 = \phi - \gamma + \epsilon_4$,
where ϵ_i are uncorrelated having mean 0 and variance σ^2.
(a) Show that $\gamma - \phi$ is estimable. What is its BLUE?
(b) Find the residual sum of squares. What is its degrees of freedom?

(10) 6. Show that the square of the multiple correlation coefficient between the response variable and the predictor variables is equal to the proportion of variability in the response variable which can be explained by linear regression on the predictor variables.