

**INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE**  
**B.MATH - Third Year, 2017-18**

**Statistics - III, Backpaper Examination, January 4, 2018**

**Time: 3 Hours**

**Total Marks: 50**

**1.** Let  $\mathbf{Y} \sim N_n(\mathbf{0}, \sigma^2 I_n)$ . Find the conditional distribution of  $\mathbf{Y}'\mathbf{Y}$  given  $\mathbf{a}'\mathbf{Y} = 0$  where  $\mathbf{a}$  is a non-zero constant vector. [10]

**2.** Consider the model  $\mathbf{Y} = \mathbf{X}\beta + \epsilon$ , where  $\mathbf{X}_{n \times p}$  has  $\mathbf{1}$  as its first column and  $\epsilon \sim N_n(\mathbf{0}, \sigma^2 I_n)$ .

(a) If  $\hat{\beta}$  is the least squares estimator of  $\beta$ , show that  $(\hat{\beta} - \beta)' \mathbf{X}' \mathbf{X} (\hat{\beta} - \beta)$  is distributed independently of the residual sum of squares.

(b) Consider the case when there is only one regressor,  $X_1$ . When do we have independence of  $\hat{\beta}_0$  and  $\hat{\beta}_1$ ?

(c) Find the maximum likelihood estimator of  $\sigma^2$ . Is it unbiased? [10]

**3.** Consider the following model:

$$y_1 = \theta + \gamma + \epsilon_1$$

$$y_2 = \theta + \phi + \epsilon_2$$

$$y_3 = 2\theta + \phi + \gamma + \epsilon_3$$

$$y_4 = \phi - \gamma + \epsilon_4,$$

where  $\epsilon_i$  are uncorrelated having mean 0 and variance  $\sigma^2$ .

(a) Show that  $\gamma - \phi$  is estimable. What is its BLUE?

(b) Find the residual sum of squares. What is its degrees of freedom? [10]

**4.** Let  $\mathbf{X} = (X_1, X_2, X_3, X_4)'$  have mean  $\mathbf{0}$  and covariance matrix  $\sigma^2 \{(1 - a^2)I_4 + a^2 \mathbf{1}\mathbf{1}'\}$ , for some  $0 < |a| < 1$  and where  $\mathbf{1}$  is the vector with all elements equal to 1. Find the partial correlations,  $\rho_{12.3}$  and  $\rho_{12.34}$ . [10]

**5.** What is a 2-factor 2-way ANOVA model? Derive its ANOVA table. Give an expression for the coefficient of determination for such a model. [10]