INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, Statistics - III, Backpaper Examination

1. Let $\mathbf{Y} \sim N_p(\mathbf{0}, \sigma^2 I_p)$. Find the conditional distribution of $\mathbf{Y}'\mathbf{Y}$ given $\mathbf{a}'\mathbf{Y} = 0$ where $\mathbf{a}$ is a non-zero constant vector. [10]

2. Consider the model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\mathbf{X}_{n\times p}$ has rank $r \leq p$ and $\epsilon \sim N_n(\mathbf{0}, \sigma^2 I_n)$.
(a) If $\hat{\beta}$ is any least squares estimator of β, show that $(\hat{\beta} - \beta)'\mathbf{X}'\mathbf{X}(\hat{\beta} - \beta)$ is distributed independently of the residual sum of squares.
(b) Find the maximum likelihood estimator of σ^2. Is it unbiased? [10]

3. Consider the following model:

$$\begin{aligned} y_1 &= \theta + \gamma + \epsilon_1 \\ y_2 &= \theta + \phi + \epsilon_2 \\ y_3 &= 2\theta + \phi + \gamma + \epsilon_3 \\ y_4 &= \phi - \gamma + \epsilon_4, \end{aligned}$$

where ϵ_i are uncorrelated having mean 0 and variance σ^2.
(a) Show that $\gamma - \phi$ is estimable. What is its BLUE?
(b) Find the residual sum of squares. What is its degrees of freedom? [10]

4. Let $\mathbf{X} = (X_1, X_2, X_3, X_4)'$ have mean $\mathbf{0}$ and covariance matrix $\sigma^2 \left\{(1 - a^2)I_4 + a^2\mathbf{1}\mathbf{1}'\right\}$, for some $0 < |a| < 1$ and where $\mathbf{1}$ is the vector with all elements equal to 1. Find the partial correlations, $\rho_{12.3}$ and $\rho_{12.34}$. [10]

5. What is a 2-factor 2-way ANOVA model? Derive its AVOVA table. Give an expression for the coefficient of determination for such a model. [10]