INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, 2019-20
Statistics - III, Semesteral Examination, November 18, 2019
Marks are shown in square brackets. **Total Marks: 50**

1. Let $Z_i, 1 \leq i \leq 4$ be independent $N(\mu, \sigma^2)$ random variables. Define $X_1 = Z_1$, $X_2 = Z_1 + Z_2$, $X_3 = Z_1 + Z_3$ and $X_4 = Z_2 + Z_4$. Let $\mathbf{X} = (X_1, \ldots, X_4)'$.
(a) Find the probability distribution of $\mathbf{X}$.
(b) Find the partial correlation coefficients $\rho_{12.3}$ and $\rho_{12.34}$ (between elements of $\mathbf{X}$). [10]

2. Consider the model:
$y_1 = \theta + \gamma + \epsilon_1$
$y_2 = \theta - \phi + \epsilon_2$
$y_3 = 2\theta - \phi + \gamma + \epsilon_3$
$y_4 = \phi + \gamma + \epsilon_4$,
where ϵ_i are uncorrelated having mean 0 and variance σ^2.
(a) Show that $\theta + 2\gamma + \phi$ is estimable. What is its BLUE?
(b) What is the degrees of freedom of the residual sum of squares? [14]

3. (a) Define the multiple correlation coefficient and the coefficient of determination. Explain how they are related to each other.
(b) What is a q-q plot or a normal probability plot? How is it useful in linear regression?
(c) How is Bonferroni inequality used for multiple comparisons in the one-way classification model? [12]

4. Consider the Gauss-Markov model, $\mathbf{Y} = X\beta + \epsilon$, where $\epsilon \sim N_n(\mathbf{0}, \sigma^2 I_n)$ and $X_{n\times p}$ has rank $r \leq p$. Consider testing $H_0 : A\beta = 0$, where $A_{q\times p}$ has rank q and $A\beta$ is a collection of estimable linear functions of β. Let $\hat{\beta}$ be a least squares solution of β and $(X'X)^-$ be a generalized inverse of $X'X$. Let $\hat{\beta}_0$ be a least squares solution of β under the constraints given by H_0.
(a) Show that $A(X'X)^-X'X(X'X)^-A' = A(X'X)^-A'$.
(b) Find the probability distribution of $A\hat{\beta}$.
(c) Find $E((A\hat{\beta})'(A(X'X)^-A')^{-1}A\hat{\beta})$.
(d) Find the distribution of $(RSS_{H_0} - RSS)/RSS$ when H_0 is true. [14]