INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, 2018-19
Statistics - III, Semesteral Examination, November 19, 2018
Marks are shown in square brackets. Total Marks: 50

1. Let $Z_i, 1 \leq i \leq 4$ be independent $N(0, \sigma^2)$ random variables. Define $X_1 = Z_1$ and $X_2 = Z_2 + Z_3$, $X_3 = Z_3 + Z_4$ and $X_4 = Z_1 + Z_2 + Z_3$. Let $\mathbf{X} = (X_1, \ldots, X_4)'$.
(a) Find the probability distribution of $\mathbf{X}$.
(b) Find the conditional distribution of (X_3, X_4) given X_2.
(c) Find the partial correlation coefficients $\rho_{12.3}$ and $\rho_{34.2}$ (between elements of $\mathbf{X}$). [12]

2. Consider the model:

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij},$$

$1 \leq i \leq 4$, $1 \leq j \leq 10$, $\sum_{i=1}^{4} \alpha_i = 0$, where ϵ_{ij} are i.i.d. $N(0, \sigma^2)$.
(a) Find the best linear unbiased estimators of μ and α_i.
(b) Construct 95% simultaneous confidence set for the vector $(\alpha_1 - \alpha_2, \alpha_2 - \alpha_3, \alpha_3 - \alpha_4, \alpha_4 - \alpha_1)$ using the method of Scheffe. Is it different from the one obtained using the method of Bonferroni? Justify. [15]

3. (a) Explain the concepts of randomization, blocks and confounding in the context of design of experiments.
(b) Suppose we want to compare two treatments. When will an experiment with matched pairs be superior to another with two independent samples? [11]

4. Consider the Gauss-Markov model, $\mathbf{Y} = X\beta + \epsilon$, where $\epsilon \sim N_n(\mathbf{0}, \sigma^2 I_n)$ and $X_{n\times p}$ has rank $r \leq p$. Suppose $T_{n\times(p-1)}$, which is formed by the first $p-1$ columns of X, has rank r also. Let B denote any generalized inverse of $T'T$.
(a) Show that $\hat{\beta} = \begin{pmatrix} BT'\mathbf{Y} \\ 0 \end{pmatrix}$ minimizes $(\mathbf{Y} - X\beta)'(\mathbf{Y} - X\beta)$.
(b) Does BLUE of β exist? Find it if it exists. [12]