**INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE**
**B.MATH - Third Year, First Semester, 2006-07**
**Statistics - III, Semesteral Examination, December 6, 2006**

**(8) 1.** Consider the model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, $i = 1, \ldots, n$, where $\epsilon_i$ are i.i.d. $N(0, \sigma^2)$. Find the joint probability distribution of $\frac{1}{n}\sum_{i=1}^{n} y_i$ and residual sum of squares.

**(12) 2.** Let $Y_1, \ldots, Y_n$ be independent random variables with unit variance, and let $X_1 = Y_1$, $X_i = Y_i - Y_{i-1}$ for $1 < i \leq n$.
(a) Find the covariance matrix of $\mathbf{X} = (X_1, X_2, \ldots, X_n)'$.
(b) Find the partial correlations $\rho_{12.3}$ and $\rho_{12.34}$ (between components of $\mathbf{X}$).

**(15) 4.** The response time in milliseconds was determined for three different types of circuits that could be used in an automatic valve shutoff mechanism. The results were the following:

| Circuit | Response time (ms) | | | | |
|---|---|---|---|---|---|
| 1 | 9 | 12 | 10 | 8 | 15 |
| 2 | 20 | 21 | 23 | 17 | 30 |
| 3 | 6 | 5 | 8 | 16 | 7 |

(a) Describe the methodology for determining whether the response times for the different circuit types significantly differ. Numerical computations are not needed.
(b) What is meant by a linear contrast in an experiment like this?
(c) What is the relation between the ANOVA null hypothesis and the hypotheses to check various linear contrasts?

**(15) 5.** Consider a completely randomized two-factor experiment.
(a) What do main effects and interactions mean in this context? Relate them to the means of cells formed by the levels of the factors.
(b) Describe how presence of these parameters can be checked.
(c) Provide an expression for the proportion of variability in the response variable explained by the two-factor model.