

**INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE**

**B.MATH - Third Year, First Semester, 2002-03**

**Statistics - III, Semesteral Examination, November 27, 2002**

(10) 1. Consider the model  $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ ,  $i = 1, \dots, n$ , where  $\epsilon_i$  are uncorrelated errors with mean 0 and variance  $\sigma^2$ . Show that the coefficient of determination in this case is the square of the sample correlation coefficient between  $y$  and  $x$ .

(10) 2. Consider the model  $\mathbf{Y} = \mathbf{X}\beta + \epsilon$ , where  $\mathbf{X}$  (which has  $(1, \dots, 1)'$  as its first column) has full column rank. Let  $\epsilon \sim N_n(0, \sigma^2 I_n)$ . Further, let  $\hat{\mathbf{Y}} = \mathbf{X}\hat{\beta}$  where  $\hat{\beta}$  is the least squares estimator of  $\beta$ . Find the joint probability distribution of  $\frac{1}{n} \sum_{i=1}^n y_i$  and  $\sum_{i=1}^n (y_i - \hat{y}_i)^2$ .

(10) 3. Consider the model  $\mathbf{Y} = \mathbf{X}\beta + \epsilon$ , where  $\mathbf{X}$  does not have full column rank, and  $\epsilon$  has mean 0 and covariance matrix  $\sigma^2 I_n$ . Let  $\hat{\beta} = (\mathbf{X}'\mathbf{X})^- \mathbf{X}'\mathbf{Y}$ , where  $(\mathbf{X}'\mathbf{X})^-$  is any generalized inverse of  $(\mathbf{X}'\mathbf{X})$ . Show that  $\hat{\beta}$  minimizes  $(\mathbf{Y} - \mathbf{X}\beta)'(\mathbf{Y} - \mathbf{X}\beta)$ .

(10) 4. Let  $\mathbf{X} = (X_1, X_2, X_3, X_4)'$  have mean 0 and covariance matrix  $\sigma^2 \{(1-a)I_4 + a\mathbf{1}\mathbf{1}'\}$ , for some  $0 < a < 1$  and where  $\mathbf{1}$  is the vector with all elements equal to 1. Find the partial correlations  $\rho_{12.3}$  and  $\rho_{12.34}$ .

(10) 5. Four chemists are asked to determine the percentage of methyl alcohol in a certain chemical compound. Each chemist makes three determinations, and the results are the following:

Chemist	Methyl Alcohol (%)		
1	84.99	84.04	84.38
2	85.15	85.13	84.88
3	84.72	84.48	85.16
4	84.20	84.10	84.55

Describe the methodology for determining whether the chemists significantly differ in their chemical analysis. Numerical computations are not needed.