

Indian Statistical Institute, Bangalore

B. Math (Hons.) First Year

First Semester - Analysis I

Back Paper Exam

Maximum marks: 100

Date: 22nd December 2023

Duration: 3 hours

Each question carries 10 marks

1. For $x, y \in \mathbb{R}$ with $x > 0$ prove that there exists $N \in \mathbb{N}$ such that $Nx > y$.
2. Find \liminf and \limsup for sequences: (a) $x_n = \cos n$; (b) $y_n = \frac{n}{2} - [\frac{n}{2}]$.
3. If $\limsup |\frac{a_{n+1}}{a_n}| < 1$, prove that $\sum a_n$ converges.
4. Suppose that the coefficients of the power series $\sum a_n x^n$ are integers, infinitely many of which are distinct from zero. Prove that the radius of convergence is at most 1.
5. Suppose $f: [a, b] \rightarrow \mathbb{R}$ is continuous. Prove that f is bounded.
6. Prove that for any continuous function $f: [0, 1] \rightarrow [0, 2]$ there exists a $x \in [0, 1]$ such that $f(x) = 2x$.
7. Prove that composite of uniformly continuous functions are uniformly continuous.
8. Suppose $f: (a, b) \rightarrow \mathbb{R}$ has a local maximum at x and is differentiable at x . Prove that $f'(x) = 0$.
9. If $|f(x) - f(y)| \leq |x - y|^3$, prove that f is constant.
10. If f is a differentiable function with bounded derivative, prove that f is uniformly continuous.