Indian Statistical Institute, Bangalore

B. Math (Hons.) Third Year

First Semester - Introduction to stochastic processes

Midterm Exam — Date: September 13, 2018

Maximum marks: 40 — Duration: 3 hours

1. Given $P = (P(i,j), i,j = 1,2,...n$ is a non-negative matrix, suppose that for some power $k \geq 1, P^k\{p_{i,j}^{(k)}\}$ is such that $P_{i,i+1}^{(k)} > 0, i = 1,2...n-1$, and $P_{n,1}^{(k)} > 0$. Show that P is irreducible; and that it may be periodic (by example). [10]

2. Consider a machine which can be in two states, 'on'and 'off'. If the machine is 'on' today, then the probability is α that it will be 'off' tomorrow. Similarly, β, is the probability of transition from 'off' state to 'on' state in one day. Denote the 'on'and 'off' states by 0 and 1 respectively. Denoting by X_n the state of the machine on day $n, (X_n)_{n\geq 0}$ is a Markov Chain with state space $S = \{0, 1\}$. Find $P(x_n = 1/x_0 = 0)$ and $\lim_{n\to\infty} P(x_n = 1/x_0 = 0)$. [5]

3. Consider the Markov chain (homogeneous) $\{X_n\}_{n=0}^{\infty}$ on state space $S = \{1, 2, 3, 4, 5, 6, 7\}$.

$$P = \begin{pmatrix} 0 & 1/2 & 0 & 0 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 0 & 0 & 1/3 & 1/3 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 0 & 1/2 \\ 1/3 & 1/3 & 0 & 0 & 0 & 1/3 & 0 \end{pmatrix}.$$

Find all communicating classes, transient states and recurrent states. Find $\lim\limits_{n\to\infty} P(X_n = i|X_o = 6) i = 1, 2, ...7$. [15]

4. Give criteria for recurrence of a state in discrete state Markov chain. Find all the recurrent and transient states in simple symmetric random walk on $\mathbb{Z}^d$, where $d \geq 1$. [10]