Indian Statistical Institute, Bangalore Centre
B.Math. (III Year) : 2012-2013
Semester I : Mid-Semestral Examination
Probability III (Stochastic Processes)

20.09.2012 Time: $2\frac{1}{2}$ hours. Maximum Marks : 80

Note: The paper carries 83 marks. Any score above 80 will be taken as 80. State clearly the results you are using in your answers.

1. (15 marks) Let $\{X_n : n \geq 0\}$ denote the unrestricted simple random walk on $\mathbb{Z}$ with $p \neq (1-p)$; here $p = P_{i,i+1}$ for all i. What is $\lim_{n\to\infty} X_n$ with probability one?

2. (10+7+8 = 25 marks) (i) Let y be a transient state for a Markov chain $\{X_n : n \geq 0\}$ on a countable state space S. Let $G(x, y)$ denote the expected number of visits to state y with $X_0 = x$. Show that $G(x, y) < \infty$, for any $x \in S$.

   (ii) Let y be as in (i) above. Show that $\lim_{n\to\infty} P_{xy}^{(n)} = 0$, for any $x \in S$.

   (iii) Using the above, show that an irreducible Markov chain on a finite state space is recurrent.

3. (15 marks) Find the stationary probability distribution for the transition probability matrix

$$\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \end{pmatrix} \end{array}$$

4. (10+8+10=28 marks) (i) Let $\{X_n\}$ be an irreducible Markov chain on a countable state space S having a stationary probability distribution π. Show that π is the unique stationary probability distribution for $\{X_n\}$.

   (ii) Let X_n be as in (i) where S has at least 2 elements. Can $\lim_{n\to\infty} X_n$ exist?

   (iii) If the assumption of irreducibility is removed in (i) above, is the assertion in (i) still valid?