MID-SEMESTER EXAMINATION
B. MATH III YEAR, II SEMESTER 2008-2009
INTRODUCTION TO STOCHASTIC PROCESSES

Max. Score:40 Time limit: 3hrs.

1. Starting from the definition of a (homogeneous) Markov Chain $\{X_n\}$ show that
$P\{X_7 = i_7, X_5 = i_5, X_4 = i_4 | X_3 = i_3, X_1 = i_1\} = p_{i_3 i_4} p_{i_4 i_5} p^{(2)}_{i_5 i_7}$ where p_{ij} and $p^{(2)}_{ij}$ are the (i, j) elements of the transition matrix P of $\{X_n\}$ and its square, respectively. [8]

2. Let $\{X_n\}$ be a Markov chain with transition matrix

$$P = \begin{bmatrix} 0.5 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0.4 & 0 & 0 & 0.6 & 0 \\ 0 & 0.2 & 0.3 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.3 & & 0.7 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 & 0 & 0.8 \end{bmatrix}.$$

Classify the states into transient, positive recurrent and null-recurrent states. Find the period of each recurrent state. Prove that the vector space $\{\pi \in \mathbb{R}^4 : \pi P = \pi\}$ is two-dimensional and find a basis for this space. [15]

3. There are two urns and a total of six balls in them. A ball is chosen at random and transferred to the other urn. This process is repeated infinitely (and independently). Let X_n be the number of balls in the first urn at time n. Find the transition matrix of the Markov chain $\{X_n\}$ as well as its stationary distribution. [8]

4. Let

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1/3 & 2/3 & 0 \\ 1/2 & 0 & 1/2 & 0 \end{bmatrix}.$$

Find $\lim_{n\to\infty} p^{(n)}_{31}$. [10]