INDIAN STATISTICAL INSTITUTE

Semestral Examination: 2019-20 (First Semester)
Bachelor of Mathematics (B. Math.) III Year
Introduction to Stchastic Processes

Teacher: Parthanil Roy

Date: 20/11/2019 **Maximum Marks: 50** **Duration: 10:00 am - 01:00 pm**

Note:

- **Please write your roll number on top of your answer paper.**
- **You may use any theorem proved or stated in the class but do not forget to quote the appropriate result.**
- **You are NOT allowed to use class notes, books, homework solutions, list of theorems, formulas etc. If you are caught using any, you will get a zero in this examination.**
- **Failing to follow the examination guidelines, copying in the examination, rowdyism or some other breach of discipline or unlawful/unethical behavior, etc. are regarded as unsatisfactory conduct. Any student caught cheating or violating examination rules will get a zero in this examination.**

1. Suppose $\{N_t\}_{t\geq 0}$ is a homogeneous Poisson process with rate α as described in the class. Fix $k, n \in \mathbb{N}$ and $0 < s_1 < s_2 < \cdots < s_k < u$. Find the conditional distribution of $\left(N_{s_1}, N_{s_2}, \ldots, N_{s_k}\right)$ given $N_u = n$. What is your answer if $n = 0$? [$9 + 1 = 10$]

2. Let $\{X_n\}_{n\geq 0}$ and $\{Y_n\}_{n\geq 0}$ be two independent Markov chains on the countable state spaces S and T, respectively with transition probability matrices P and Q, respectively. Define a new stochastic process $Z_n = (X_n, Y_n)$, $n \geq 0$.

   (a) Show that $\{Z_n\}_{n\geq 0}$ is also a Markov chain on the state space $S \times T$ and write down its transition probability matrix. *We will call this the direct product of* $\{X_n\}_{n\geq 0}$ *and* $\{Y_n\}_{n\geq 0}$. *This notion can be defined for any number of Markov chains.* [$3 + 3 = 6$]

   (b) If $\{X_n\}_{n\geq 0}$ and $\{Y_n\}_{n\geq 0}$ are both irreducible, then will the direct product also be irreducible? Please justify your answer. [4]

3. For each of the following Markov chains, we mention a state. Find whether that state is transient or null recurrent or positive recurrent. Justify your answer in each case quoting appropriate results covered in the class. [$10 \times 3 = 30$]

   (a) The state 0 in the biased nearest neighbour random walk on $\mathbb{Z}$.

   (b) The state $(0, 0, \ldots, 0)$ in the simple random walk on the group $\mathbb{Z}_2^d$.

   (c) The state $(0, 0, 0)$ in the direct product of three simple random walks on $\mathbb{Z}$.

Wish you all the best