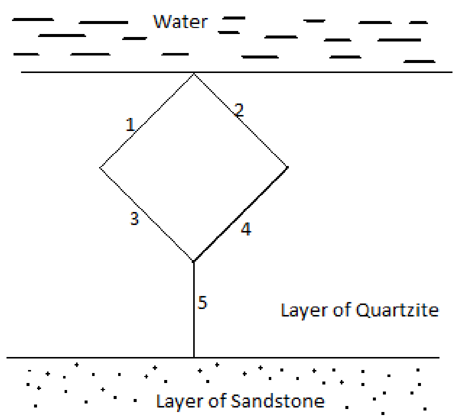
INDIAN STATISTICAL INSTITUTE
Probability Theory I: B. Math (Hons.) I
Semester I, Academic Year 2023-24
Midsem Exam

Date: 15/09/2023 **Full Marks: 50** **Duration: 3 hours**

- **Show all your work and write explanations when needed. If you are using a result stated and/or proved in class, please quote it correctly.**
- **You are NOT allowed to use class notes, books, homework solutions, list of theorems, formulas etc.**

1. Consider the following schematic diagram of a drainage network model (as described in the class), where each of Paths 1 - 5 (as shown in the figure below) behave independently of each other. Suppose that each path is open with probability $p \in (0, 1)$. Recall that water will be able to pass through a particular path (only downwards) if and only if it is open.



   (a) (10 marks) Find the (unconditional) probability that water can pass through the layer of quartzite to the layer of sandstone.
   (b) (10 marks) If it is given that Path 2 and Path 3 are both open, calculate the conditional probability that water can pass through the layer of quartzite to the layer of sandstone.
   (c) (10 marks) If it is given that exactly one of Path 2 and Path 3 is closed, calculate the conditional probability that water can pass through the layer of quartzite to the layer of sandstone.

Plese Turn Over

2. (10 marks) Suppose r ($\in \mathbb{N}$) distinguishable flags are displayed at random on n ($\in [2, r]$) distinguishable poles. Find the probability that every pole receives at least one flag.

3. (10 marks) Fix two positive integers r_1, r_2. Suppose r_1 many α's and r_2 many β's are arranged at random. For every positive interger k, compute the probability of the event that *there are exactly k many α-runs and exactly $k+1$ many β-runs*. Please also mention the range of k for which this probability is nonzero.