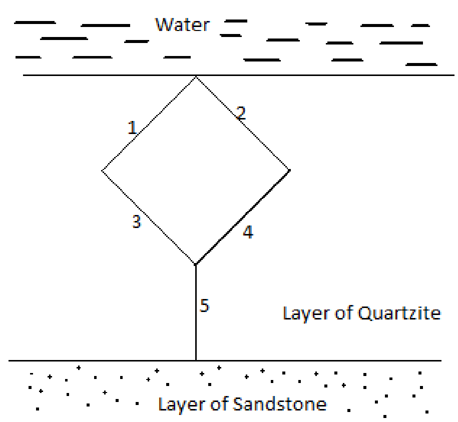
INDIAN STATISTICAL INSTITUTE
Probability Theory I: B. Math (Hons.) I
Semester I, Academic Year 2022-23
Midsem Exam

Date: 10/10/2022 **Full Marks: 50** **Duration: 3 hours**

- **Show all your work and write explanations when needed. If you are using a result stated and/or proved in class, please quote it correctly.**
- **You are NOT allowed to use class notes, books, homework solutions, list of theorems, formulas etc.**

1. Consider the following schematic diagram of a drainage network model (as described in the class), where each of Paths 1 - 5 (as shown in the figure below) behave independently of each other. Suppose that each path is open with probability $p \in (0, 1)$. Recall that water will be able to pass through a particular path (only downwards) if and only if it is open.



   (a) (10 marks) If it is given that Path 4 is open, calculate the probability that water can pass through the layer of quartzite to the layer of sandstone.

   (b) (10 marks) If it is given that Path 4 is closed, calculate the probability that water can pass through the layer of quartzite to the layer of sandstone.

Plese Turn Over

2. (10 marks) Consider Polya's Urn Scheme as described in the class (starting with b black balls and r red balls, and adding c balls at each stage). Calculate the probability that in the first three draws, exactly one black ball is drawn.

3. (10 marks) Suppose r ($\in \mathbb{N}$) distinct toys are distributed at random among n ($\in [2, r]$) children. Find the probability that every child receives at least one toy.

4. (10 marks) Fix two positive integers r_1, r_2. Suppose r_1 many α s and r_2 many β s are arranged at random. For every positive interger k, compute the probability of the event that *there are exactly k many α-runs and exactly k many β-runs.*