Indian Statistical Institute, Bangalore Centre.
End-Semester Exam : Probability 1

Instructor : Yogeshwaran D.

Date : January 25th, 2022.

Max. points : 40. **Time Limit : 2 h +30 m.**

Submit solutions via Moodle by 12.30 PM on January 25th.

Please write your name and the honesty statement below on your answer script and sign below the same. Else 2 points will be deducted.

I have not received, I have not given, nor will I give or receive, any assistance to another student taking this exam, including discussing the exam with other students. The solution to the problems are my own and I have not copied it from anywhere else. I have used only class notes, notes from TA sessions, my own notes and assignment solutions.

Answer all four questions. All questions carry 10 points.

Give necessary justifications and explanations for all your arguments. If you are citing results from the class, mention it clearly. Always define the underlying random variables and events clearly before computing anything !

1. Suppose we have 10 coins such that if the ith coin is flipped, heads will appear with probability $i/10, i = 1, 2, ..., 10$. One of the coins is randomly selected and tossed.
   (a) What is the probability that coin shows heads ? (**3**)
   (b) Suppose that the coin shows heads. What is the conditional probability that the tenth coin has been selected ? (**3**)
   (c) The coin is tossed again independently of the first toss. Suppose both the tosses are heads. What is the conditional probability that the tenth coin has been selected ? (**4**)

2. A pack of cards consists of n distinct cards (label them $1, \ldots, n$) being repeated s many times i.e., there are sn cards in total. Suppose a random sample of size r ($r \geq n$ is

drawn from the pack of cards without replacement what is the probability that at least one card of each label $1, \ldots, n$ is present in the sample ?

3. A jar contains n chips. Suppose a student successively draws a chip at random from the jar, each time replacing the one drawn before drawing another. The process continues until the student draws a chip that has been previously drawn. Let X denote the number of draws. Compute the pmf and expectation of X.

4. Let Z be a standard normal random variable.

   (a) Show that $\mathbb{E}\big[Z^{n+1}\big] = n\mathbb{E}\big[Z^{n-1}\big]$ for all $n \geq 1$ and compute $\mathbb{E}[Z^n]$ for all $n \geq 1$. **(5)**

   (b) Find the pdf of the random variable $Y = e^{aZ+b}$. **(5)**