ISI – Bangalore Center – B Math - Physics IV – Mid Term Exam
Date: March 7, 2014 Duration of Exam: 3 hours
Total marks:40

Answer any 5 from Q1 to Q6 AND either Q6 or Q7.

Please note:
a. The spacetime metric is (1, -1, -1, -1). A generic 4-vector will be denoted by $x^\mu = (x^0, x^1, x^2, x^3)$ where x^0 is the zero or the time component; e.g. (ct, x, y, z) is a 4-vector

b. Unless stated otherwise, all Lorentz transformations are homogeneous, proper, and orthochronous.

Q1. [1+1+1+2=5] Consider events at spacetime points A, B, and C whose coordinates in a frame S is given by A: (ct=20,x=10, y=50, z=30), B: (ct=30, 0,0,10), and C: (ct= - 10,0,0,0).

Determine if the following three statements are true or false:
1a.) There exists a frame in which A and B are simultaneous
1b.) There exists a frame in which A and C are simultaneous
1c.) There exists a frame in which B and C are simultaneous

1d.) Let S' be another frame moving with respect to the frame S with velocity v such that $\frac{\vec{v}}{c} = (\frac{\sqrt{5}}{10}, \frac{\sqrt{11}}{5}, \frac{\sqrt{15}}{10})$. If there is a clock in S at the location (50, 70, 10), according to an observer in S' by what factor will it run slower? Justify your answer.
==================================

Q2. [5] Two events occur at spacetime points whose coordinates are given in frame S as A: (ct=1000, 500, 90, 2000) and B: (ct=0,400,90,1000). Observers in S claim that "A happens after B".

Explain why in this specific situation the statement "A happens after B" is not a relativistically invariant statement. Your reasoning must be quantitative.

==================================

Q3. [3+2=5] A ray of light propagating in the xy plane, making an angle θ with +x axis as seen in frame S. What will be the components of velocity of this ray as measured from a frame S' moving with velocity v with respect to S in the +x direction. Show that the magnitude of this velocity will be the same but the angle of propagation in the S prime frame will be given by

$$\cos\theta' = \frac{\cos\theta - \beta}{1 - \beta\cos\theta}$$

Q4. [5] Let A and B be simultaneous events in frame S occurring along the x axis. Show that they remain simultaneous in $S^{'}$ moving with velocity (0, v, 0) w.r.t. to S.

================================

Q5. [3+2=5] A body of rest mass m_0 moving at speed v approaches an identical body at rest. Find V the speed of a frame in which the total three-momentum is zero.

What are the limiting values of V as v/c tends to zero and also when v/c tends to ∞?

=================================

Q6. [5] Show that if the applied force is parallel to the velocity $\vec{u}$, then equation of motion reduced to $F = m_0\gamma^3 \frac{du}{dt}$ where m_0 is the rest mass, and F, u are magnitudes of the force and velocity respectively. (Recall that $\vec{F} = \frac{d}{dt}(m\vec{u})$ where $m = m_0\gamma$)

=================================

Q7. [8+4+3=15]

a.) Show that the following matrix represents a Lorentz transformation.

$$\begin{pmatrix} 2 & 1 & \sqrt{2} & 0 \\ \sqrt{2} & \sqrt{2} & 1 & 0 \\ 1 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b.) Explain why the Lorentz transformation above is NOT a pure boost but can be considered as a pure boost followed by a rotation of the frames and determine the axis of rotation.

c.) Assume that two frames S and $S^{'}$ are connected by the above transformation. If a rod of length L is lying at rest in $S^{'}$ ALONG its direction of motion w.r.t. to S, what is its length as measured in S? Justify your answer.

================================

Q8. [6+9=15] Let K_1 be and K_2 be the matrices corresponding to the generators of boost in x and y directions. Show that

a.) the matrix $e^{-\beta (K_1+K_2)}$ is a Lorentz transformation and is a pure Boost

b.) the matrix $e^{-\beta K_1} e^{-\beta K_2}$ is a Lorentz transformation but not a pure boost

where β is a number not necessarily small.

(Hint: use symmetry properties of boost generators and for b.) expand in powers of β to check if it is a boost.)