

**Physics IV**  
**ISI B.Math**  
**Mid Semestral Exam: March 12, 2013**

Total Marks: 70. Time: Three Hours.  
Answer ALL questions

**NOTE:** Unless otherwise stated, the four-vector  $x^\mu$  stands for  $(x^0 = ct, x^1, x^2, x^3)$ . The norm of the four vector is  $x^2 = \eta_{\mu\nu}x^\mu x^\nu$  where  $\eta_{00} = -\eta_{11} = -\eta_{22} = -\eta_{33} = 1$  and  $\eta_{\mu\nu} = 0$  for  $\mu \neq \nu$

**Question 1. [2+1+1+6]**

1a.) What are the four components of the velocity four-vector  $U^\mu$  in terms of the velocity of a moving particle in that frame?

1b.) What is the norm of the velocity four-vector defined above?

1c.) How do components of the velocity four-vector transform under an arbitrary Lorentz transformation  $L$ ?

1d.) Either using the above or directly from Lorentz transformation formula, derive the rule for transformation of ordinary velocities  $\vec{u}'$  and  $\vec{u}$  where  $u^i = \frac{dx^i}{dt}$ ,  $i = 1, 2, 3$  etc. under a boost  $v$  in the  $+x$  direction.

**Question 2. [4+6]**

Two rods  $AB$  and  $CD$  of equal rest length  $L$  are moving in opposite directions along the  $x$  axis with equal speed  $v$  as seen in the lab frame.

a.) What is the time elapsed for them to cross each other in the lab frame?

b.) What is the time elapsed for them to cross each other in the frame of the rods?

**Question 3 [2+2+3+3+5]**

The frame  $S'$  is moving in the  $-x$  direction with respect to frame  $S$  with speed  $v$ . In  $S$ , a rod  $AB$  of length  $L$  is at rest between  $x = 0$  and  $x = L$ . In  $S'$ , a rod  $CD$  of length  $L$  is at rest between  $x' = 0$  and  $x' = L$ . The frames are synchronized so that  $x = x' = 0$  when  $t = t' = 0$ .

- a.) Draw the world lines of the rods.
- b.) Label the events AC, AD, BC, BD on the diagram.
- c.) Arrange the events in temporal sequence in the  $S$  frame.
- d.) Will the same temporal sequence hold in  $S'$ ? If yes, explain why. If not, rearrange then in sequential order in  $S$  prime.
- e.) Calculate the time it takes for the rods to pass each other completely in  $S$  frame and in  $S'$  frame.

**Question 4. [3x5]**

Prove or find a counter example of

- a.) The sum of two time like vectors is a time like vector
- b.) A vector that is orthogonal to a time like vector is a space like vector.
- c.) There does not exist a Lorentz frame in which two events  $(1,2,0,0)$  and  $(3,6,0,0)$  are simultaneous

**Question 5. [10]**

$S'$  is moving with respect to  $S$  with velocity  $\frac{c}{2}(\vec{i} + \vec{j})$ . Determine ALL the elements of the Lorentz transformation matrix  $L$  that connects these two frames.

**Question 6.** [10]

If space time were 5 dimensional with one time and 4 space dimensions and a Lorentz transformation is a transformation that preserves the norm  $(x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 - (x^4)^2$ , write explicitly ALL the generators of the Lorentz group in canonical form.