Physics IV
ISI B.Math
Backpaper Exam : June 10,2019

Total Marks: 60
Time : 3 hours
Answer all questions

1. (**Marks : 4 + 3 + 3 = 10**)

A train of proper length L and speed $\frac{3c}{5}$ approaches a tunnel of length L. At the moment the front of the train enters the tunnel, a person leaves the front of the train and walks towards the back. She arrives at the back of the train right when it(the back) leaves the tunnel
(a) How much time does this take in the ground frame?

(b) What is the person's speed with respect to the ground ?

(c) How much time elapses on the person's watch ?

2. (**Marks : 5 + 5 + 5 = 15**)

(a) Show that an electron and a positron (a particle with the same mass as the electron but equal and opposite charge) cannot combine to produce only *one* photon.

(b) Suppose that the four vector X is timelike or null. Show that if $X^0 > 0$ in some inertial coordinate system, then $X^0 > 0$ in every inertial coordinate system.

(c) A non accelerating observer O has four velocity U. Show that O concludes that two events A and B are simultaneous if and only if the displacement four vector X from A to B is orthogonal to U, i.e if $U^\mu X_\mu = 0$

3. (**Marks : 2 + 8 = 10**)

The Hamiltonian operator for a two state system (the vector space is spanned by the ket vectors $|1>$ and $|2>$) is given by

$$H = a(|1><1| - |2><2| + |1><2| + |2><1|)$$

where a is a number with dimensions of energy.

(a) Show that the Hamiltonian is Hermitian.

(b) Find the energy eigenvalues and the corresponding energy eigenkets (as linear combinations of $|1>$ and $|2>$)

4. (**Marks : 2 + 8 = 10**)

A particle in the infinite square well potential($V(x) = 0, 0 \leq x \leq a$, $V(x) = \infty$ otherwise) has the initial wave function $\Psi(x,0) = Ax(a-x)$,$0 \leq x \leq a$. Outside the well , of course $\Psi(x,0) = 0$.

(a) Normalize $\Psi(x, 0)$

(b) Find $\Psi(x, t)$

5. (**Marks : 2 + 5 + 6 + 2 = 15**)

The annihilation (creation) operator for the harmonic oscillator in 1-d is defined as $\hat{a} = \sqrt{\frac{m\omega_0}{2\hbar}}\left(\hat{x} + i\frac{\hat{p}}{m\omega_0}\right)$ where m is the mass and ω_0 is the angular frequency of the harmonic oscillator

(a) Express the Hamiltonian operator for the Harmonic Oscillator in terms of a and its Hermitian conjugate $a^\dagger$

(b) Show that if ψ is a solution of the time independent Schrodinger equation with energy E, $a\psi$ is a solution with energy $E - \hbar\omega$.

(c) Using (b) argue that there must exist a state of lowest energy $\psi_0(x)$ and determine its explicit form.

(d) Find the expectation value of position and momentum in the state determined in (c).