ISI – Bangalore Center – B Math - Physics IV – Backpaper Exam
Date: 11 June 2018. Duration of Exam: 3 hours
Total marks: 90

Q1. [Total Marks: 4+6+2+3=15]

A train with proper length L moves at a speed 5c/13 with respect to the ground. A ball is thrown from the back of the train to the front. The speed of the ball with respect to the train is c/3.

a.)Show by using the velocity addition theorem that the speed of the ball relative to the ground is $7c/11$.

b.)Using Lorentz transformations or otherwise find the time the ball takes to reach the front and how far it travels as viewed by someone on the ground.

c.) Determine if the interval connecting the events of the ball leaving the back of the train and the ball reaching the front of the train is time like or space like.

d.) Calculate the four velocity associated with the ball in any of these two frames and show that it is a unit time like vector.

Q2. [Total Marks: 2+2+6+5=15]

a.) Write the components of the four momentum of a particle in terms of its four velocity components.

b.) Show that the four momentum of a particle is a future oriented time like vector.

c.) Show that the sum of four momenta of two particles is a future oriented time like vector.

d.) Using the above results (or otherwise) show that an electron positron pair can not annihilate into a single photon. You can assume conservation of total four momentum in this process. Recall that a photon has zero rest mass and that an electron has the same rest mass as that of a positron.

Q3. [Total Marks: 5+5+5+5=20]

a.) Show that $\frac{d}{dt}\int \psi_1^*(x,t)\psi_2(x,t)dx = 0$ where $\psi_1(x,t)$, $\psi_2(x,t)$ are any two normalizable solutions of the Schrödinger equation

b.) Show that it is always possible to find a real solution to the one dimensional time independent Schrödinger equation.

c.) If $V(x)$ is an even function of x, the corresponding solution $\psi(x)$ of the time independent Schrödinger equation can always be taken to be either an even or odd function of x.

d.) If the operator $\hat{Q}$ representing a physical observable has no explicit time dependence then show that

$\frac{d}{dt} < \hat{Q} > = \frac{i}{\hbar} < [H, \hat{Q}] >$ where H is the Hamiltonian operator.

Q4.[Total Marks: 6+4+5+5=20]

a.) Solve the time independent Schrödinger equation for a particle of mass m in an infinite square well of width L ($V = 0$ for $0<x<L$, infinite otherwise) to find the normalized wave functions for the energy eigenstates.

b.) Show that the energies of these eigentstates are given by $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$ where n = 1,2,3….

c.) Suppose wave function at time t=0 is given by $\frac{1}{\sqrt{2}}[\psi_1(x) + \psi_2(x)]$ where $\psi_1(x), \psi_2(x)$ are eigenstates with energy E_1 and E_2. Determine $\psi(x,t)$.What is the probability of measuring energy E_3 at any time t?

d.) Show that $|\psi(x,t)|^2$ oscillates in time and determine the frequency of oscillation.

Q5. [Total Marks:5+5+10=20]

A quantum mechanical harmonic oscillator in one dimension is described by the Hamiltonian: $H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2 x^2$. Consider the operator $a_- = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$ and its Hermitian conjugate a_+.

a.) Show that the eigenvalues of the operator a_+a_- must be nonnegative real number. Hence show that the lowest energy eigenvalue has to be at least equal to or greater than $\hbar\omega/2$.

b.) Let $|n>$ be a normalized eigenstates of a_+a_- with eigenvalue *n*. Using that fact that $a_+a_- |n>= n|n>$, $a_- |n>= \sqrt{n}\,|n-1>$ and $a_+ |n>= \sqrt{n+1}\,|n+1>$ show that $<n|x|n>=<n|p|n>=0$

c.) Show that in the $|n>$ state, $\Delta x \Delta p = \hbar(n+\frac{1}{2})$