Physics IV
ISI B.Math
Backpaper Exam : June 5,2017

Total Marks: 100
Time : 3 hours
Answer all questions

1. (**Marks: 10 + 5 + 5 = 20**)

(a) Show that a Lorentz boost in the x direction with speed v_1 followed by a Lorentz boost in the x direction with speed v_2 results in a Lorentz boost along the x direction . Find the speed u of the resultant Lorentz transformation in terms of v_1 and v_2.

(b) A and B both start at the origin and simultaneously head off in opposite directions at speed $\frac{3c}{5}$ with respect to the ground. A moves to the right and B moves to the left. Consider a mark on the ground at $x = L$. As viewed in the ground frame, A and B are a distance $2L$ apart when A passes this mark. As viewed by A, how far away is B when A coincides with the mark ?

(c) Show that if one of the components of a four vector is zero in every inertial frame, then all four components are zero in every inertial frame.

2. (**Marks : 5 + 15 = 20**)

(a) Write down a relativistic expression for the kinetic energy of a particle of mass m and speed v. Show that it reduces to the usual Newtonian expression in the appropriate limit.

(b) A photon collides with a stationary electron. If the photon scatters at an angle θ with respect to the original direction, show that the resulting wavelength λ' is given in terms of the original wavelength λ , by

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$$

where m is the mass of the electron. Note: The energy of a photon is $E = h\nu = \frac{hc}{\lambda}$

3. (**Marks : = 5 + 5 + 5 + 5 = 20**)

(a) Show that $\frac{d}{dt}\int_{-\infty}^{\infty}\Psi_1^*\Psi_2 dx = 0$ where Ψ_1 and Ψ_2 are any two normalizable solutions of the Schrödinger equation.

(b) Show that it is always possible to find a real solution $\psi(x)$ to the one dimensional time independent Schrödinger equation.

(c) If $V(x)$ is an even function of x, the corresponding solution $\psi(x)$ of the time independent Schrödinger equation can always be taken to be either an even or odd function of x.

(d) If the operator $\hat{Q}$ representing an observable Q has no explicit time dependence, show

$$\frac{d}{dt} < \hat{Q} >= \frac{i}{\hbar} < [\hat{H}, \hat{Q}] >$$

where $\hat{H}$ is the Hamiltonian operator.

4. (**Marks : 10 + 3 + 7 = 20**)

A particle mass m moves under the influence of the potential $V(x) = 0$ if $0 \leq x \leq a$ and ∞ otherwise.

(a) Solve the time independent Schrödinger equation for this potential and find the stationary states $\psi_n(x)$ and their corresponding energies E_n.

(b) If the initial state is given by $\Psi(x, 0) = A[\psi_1(x) + \psi_2(x)]$, find A.

(c) Find $< x >$ in the state $\Psi(x, t)$ and show that it oscillates in time. What is the amplitude and angular frequency of the oscillation ?

5. (**Marks : 2 + 3 + 4 + 6 + 5 = 20**)

A one dimensional harmonic oscillator of mass m has potential energy $V(x) = \frac{1}{2}m\omega^2x^2$.
Consider the operators $a = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$ and $a^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x - ip)$
It is given that $a^\dagger\psi_n = \sqrt{n+1}\psi_{n+1}$ and $a\psi_n = \sqrt{n}\psi_{n-1}$, where ψ_n is a solution of the time independent Schrödinger equation with energy $E_n = (n + \frac{1}{2})\hbar\omega$

(a) Evaluate $[a, a^\dagger]$
(b) Show that a lowest energy ground state exists such that $a\psi_0 = 0$ and that the allowed values of n are non negative integers.
(c) Find the normalized ground state wave function $\psi_0(x)$ and the first excited state $\psi_1(x)$
(d) Find the expectation values $< T >$ and $< V >$ of the kinetic energy and the potential energy respectively when the particle is in an energy eigenstate ψ_n and check that $< T > = < V >$.