ISI – Bangalore Center – B Math - Physics IV – Backpaper Exam
Date: 23 May 2016. Duration of Exam: 3 hours
Total marks: 50

Answer ANY Five Questions

Q1. [Total Marks: 5+5=10]

A muon decays spontaneously and the time to decay is 2×10^{-6} seconds in its own rest frame. Suppose that a muon is created at a height of 50 km above the earth's surface and it is moving towards the earth at a speed of 0.99998c.

a.) Calculate the components of the four vector $\frac{dx^{\mu}}{ds}$ for the muon in the earth frame.

[Note that $x^{\mu} = (ct, x, y, z)$ and $ds = \sqrt{(cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2}$

What are the components of the four vector $\frac{dx^{\mu}}{ds}$ in the muon frame of reference?

What is the norm of the four vector $\frac{dx^{\mu}}{ds}$?

b.) Assuming that the muon will not collide with anything else during its journey, determine if it will reach the earth before decaying or not. [You can do this either in the muon rest frame or in the earth frame, but you need to specify the frame you are using for the calculation.]

Q2. [Total Marks: 6+4=10]

a.) A train with proper length L moves at a speed 5c/13 with respect to the ground. A ball is thrown from the back of the train to the front. The speed of the ball with respect to the train is c/3. As viewed by someone on the ground, how much time does the ball take to reach the front and how far does it travel?

b.) In the above problem verify that the interval $c(\Delta t)^2 - (\Delta x)^2$ between the two events (of the ball being thrown at the back of the train and it reaching the front end] is invariant as measured by the two frames.

Q3. [Total Marks: 5+5=10]

a.) Consider the process of collision of two particles. Suppose the total four momentum is found to be conserved in the lab frame. Show that the total four momentum will be conserved in every frame related to the lab frame by a Lorentz transformation. [Hint: A zero vector remains a zero vector under a linear transformation]

b.) A particle of rest mass m is initially at rest and decays into a photon and another particle of rest mass m- δ. Show that the photon energy is $\delta\left(1-\frac{\delta}{2m}\right)$. You can set c=1 for convenience.

Q4. [Total Marks: 3+3+4=10]

Consider a quantum mechanical system in one dimension. Assume that the wave functions used below are normalizable and go to zero "sufficiently fast"

a.) Show that the momentum operator $p=-ih\frac{d}{dx}$ is Hermitian.

b.) Show that $<p^2>=<\psi|p^2|\psi>$ is always positive.

c.) Show that the norm of the wave function $\int_{-\infty}^{\infty}\psi^*(x,t)\psi(x,t)dx$ is independent of time.

Q5. [Total Marks: 4+6=10]

Consider the bound states of a one dimensional quantum system in which V(x)=V(-x).

a.) Using the fact that the bounded energy eigenstates in 1 dimension are non degenerate, show that the energy eigenstates $\psi_E(x)$ must be either even or odd function of x.

b.) Consider a particle in one dimension in a potential $V(x)=-V_0, -a<x<a, V_0>0$ and zero elsewhere. Show that the odd energy eigenstates are given by

$$\sqrt{\frac{-2mE}{\hbar^2}}=-\sqrt{\frac{2m(V_0-E)}{\hbar^2}}\cot\left(\sqrt{\frac{2m(V_0-E)}{\hbar^2}}a\right)$$

Show that V_0 must be greater than a critical value for an odd eigenstate to exist.

Q6. [Total Marks: 3+4+3 =10]

Consider a quantum mechanical harmonic oscillator described by the Hamiltonian:

$$H=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}m\omega^2x^2.$$

a.) Show that the energy eigenvalues of a harmonic oscillator must be greater than zero.

b.) Consider the operator $a_- = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$.

Show that $[a_-, H] = \hbar\omega$, hence show that there must be a state ψ_0, such that $a_-\psi_0 = 0$

c.) Determine the functional form of ψ_0 by solving $a_-\psi_0 = 0$