Physics IV
Indian Statistical Institute, Bangalore. B.Math III Year
Final Exam: May 3, 2013

Total Marks: 70. Time: Three Hours.

NOTE: Please note that Question 5 needs to be answered in the question sheet and submitted along with the rest of the answer sheets.

Question 1. [15]
The Hamiltonian of a planar rigid body with moment of inertia I rotating freely in the xy plane is given by $H = (-\frac{\hbar^2}{2I})\frac{\partial^2}{\partial\phi^2}$ where ϕ is the angle of rotation.

a.) Show that the allowed energy levels of the system are either zero or multiples of $\frac{\hbar^2}{2I}$. What are the corresponding energy eigenstates?

b.) If at time $t = 0$ the initial wave function $\psi(x, t = 0) = A\sin^2\phi$, where A is the normalization constant, what is $\psi(x,t)$ for $t > 0$?

Question 2. [15]
Consider a 1-dim quantum particle of mass m in a potential $V(x) = \infty$ for $x < 0$, $V(x) = 0$ for $0 \leq x \leq 0$, and $V(x) = V_0$ for $x \geq a$.

a.) Show that the bound state energies E, $(0 \leq E \leq V_0)$ are given by the equation
$\tan\frac{\sqrt{2mE}a}{\hbar} = -\sqrt{\frac{E}{V_0 - E}}$

b.) Without solving the equation further, sketch the ground state wave function.

Question 3. [15]

A 1-dim harmonic oscillator is in an energy eigenstate that is described by the wave function $N(16\xi^4 - 48\xi^2 + 12)e^{-\xi^2/2}$ where N is a normalization constant, and $\xi = x(\frac{m\omega}{\hbar})^{\frac{1}{2}}$ in usual notation.

a.) Determine the energy of this state. (An answer without accompanying cogent reasoning and/or calculation will not be given credit.)

b.) Determine the wave function for the next higher energy eigenstate as a function of ξ. You need not find the value of the normalization constant.

[Potentially useful formula $a = \frac{1}{\sqrt{2m\hbar\omega}}(m\omega x + ip), p = -i\hbar\frac{\partial}{\partial x}$]

Answer Either Question 4a OR 4b.

Question 4a[10]

a.) Show that for any 3-dim spherically symmetric potential, $\frac{d}{dt} < L_x >= 0$.

b.) What is the value of $< l, m|L_x|l, m >$ where $|l, m >$ is an eigenstate of L^2 and L_z?

Question 4b[10]

An object of rest mass m_1 moving with three momentum $\vec{p_1}$ collides and coalesces with a stationary object with rest mass m_2. Show that the rest mass M of the compound object thus formed is given by

$$M^2c^2 = m_1^2c^2 + m_2^2c^2 + 2m_2(|p_1|^2c^2 + m_1^2c^4)^{1/2}$$

WRITE YOUR NAME HERE IN BLOCK LETTERS

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PLEASE WRITE YOUR ANSWERS IN THIS SHEET AND SUBMIT ALONG WITH OTHER SHEETS. Use other sheets if you need to do rough work, but you need not submit those rough works

Question 5.[1+2+2+2+3+3+2=15]

5a.) Among the operators x, y, p_z, p_y which pairs do NOT commute? Write your answer in the space below.

Please CIRCLE the right answer in 5b, c, and d

5b.) What is the Hermitian Conjugate of $(x + \frac{\partial}{\partial x})$?

(i) $i(x + \frac{\partial}{\partial x})$ (ii) $(x + i\frac{\partial}{\partial x})$ (iii) $(x - i\frac{\partial}{\partial x})$ (iv) $(x - \frac{\partial}{\partial x})$

5c.)For 1-dim harmonic oscillator,the matrix element $< n \mid \frac{1}{2}m\omega^2 x^2 \mid n+1 >$ is

(i) $n(n+1)\frac{\hbar\omega}{2}$ (ii) $(2n+1)\frac{\hbar\omega}{2}$ (iii) zero (iv) none of these

5d.) An electron confined to the ground state in an one dimensional box of width $10^{-10}m$ has energy 38 ev. A photon hits the electron and moves it to the NEXT higher energy level. What minimum energy must this photon have?

(i) 114ev (ii) 38ev (iii) 19ev (iv) 304ev