**Physics IV**
**ISI B.Math**
**End Semestral Exam : April 27, 2011**

Total Marks: 75. Time: 3 hours. Answer **any FIVE** questions

**Question 1. Total Marks:15**
Consider a particle in three dimensions and two normalized energy eigenfunctions $\psi_1(x)$, $\psi_2(x)$ corresponding to eigenvalues $E_1 \neq E_2$. Assume that the eigenfunctions vanish outside two non-overlapping regions $\Omega_1(x)$ and $\Omega_2(x)$ respectively.
(a) Show that if the particle is initially in region $\Omega_1(x)$ then it will stay there forever.
(b) If, initially, the particle is in the state with wave function
$\psi_1(x,0) = \frac{1}{\sqrt{2}}[\psi_1(x) + \psi_2(x)]$
show that the probability density $|\psi(x,t)|^2$ is independent of time.
(c) Now assume that the two regions $\Omega_1(x)$ and $\Omega_2(x)$ overlap partially. Starting with the initial wave function of case (b), show that the probability density is a periodic function of time.

**Question 2. Total Marks:15**
Assume that a particle in a 3-d central potential is in a state with an orbital angular momentum with z component $\hbar m$ and square magnitude $\hbar^2 l(l+1)$
(a) What is the value of $< L_z >$?

(a) What is the value of $< L_x >$?

(b) Show that $< L_x^2 >= \frac{1}{2}[\hbar^2 l(l+1) - \hbar^2 m^2]$

(c) Based on the above what will be the values of $< L_y >$ and $< L_y^2 >$? Justify your answers.

**Question 3. Total Marks:15**
A one dimensional quantum spring that can be stretched but cannot be compressed may be represented by the potential

$V(x) = \frac{1}{2}m\omega^2 x^2$ for $x > 0$ and $\infty$ for $x < 0$

(a) What is the value of the wave function in the region $x < 0$?
(b) What is the behaviour of the wave function for large values of $x$?
(c) Determine all the energy eigenvalues of the system.

Justify your answers.[Hint:This problem requires very little actual computation. You can use results from the standard harmonic oscillator]

**Question 4. Total Marks:15**
Take a two dimensional quantum system. Assume that $H = \frac{1}{2m}[p_x^2 + p_y^2] + V(r)$

Consider the operator $R = -i\hbar(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x})$
(a) Show that $< R >$ is time independent. Hint: Calculate $[R, H]$
(b) Show that $R$ has real eigenvalues. Calculate the eigenstates and eigenvalues of R. Hint: use radial coordinates.
(c) What conditions must eigenvalues of $R$ satisfy?

**Question 5. Total Marks:15**

(a) Prove that if any one of the components of a Lorentz 4-vector is zero in all inertial frames, then the vector must be a null vector.

(b) Use this result to show that if any of the components of a total 4-momentum of a closed system is conserved in all frames, all components of the 4-momentum must be conserved.

**Question 6. Total Marks:15**

(a) Prove that the 4x4 Lorentz transformation matrix connecting two inertial frames which have parallel coordinate axes but are moving with a relative velocity $\pm\overrightarrow{v}$ is symmetric.

(b) Use this result to determine if composition of two boosts is another boost or not.