Physics IV Final Exam

Maximum Marks : 100 Duration: 3 hours

Answer all questions

$x_2 = \gamma\,(x_1 - vt_1)$

$t_2 = \gamma\,(t_1 - \frac{\beta}{c}x_1)$

$\beta = v/c$

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$

1. a) State the postulates of special theory of relativity. 5

b) The coordinate systems S_1 and S_2 move along the x-axis of a reference frame S with velocities V_1 and V_2 respectively (referred to S). The time measured in S for the hand of a clock in S_1 to go around once, is t. What is the time interval in S_2 for the hand to go around? 7

c) A radio wave of frequency ν is emitted by the source in the direction of a rocket ship traveling towards the source with velocity v. What will be the frequency of the wave measured in the rocket ship? 8

2.

$$\rho(\lambda) = \frac{8\pi hc}{\lambda^5}\frac{1}{(e^{hc/\lambda kt}-1)}$$

a) Using Planck's radiation law, for ρ(λ) prove that the total energy density

$$\rho_{tot} = aT^4 \text{ where } a = \frac{8\pi^5 k^4}{15h^3c^2}$$

Useful integral ($\int_0^\infty \frac{x^3}{e^{x-1}}dx = \frac{\pi}{15}^4$) 10

b) The photoelectric work function W for lithium is 2.3 eV.

i. Find the threshold wavelength λ_t for the photoelectric effect. 5

ii. If UV light of λ=2000 Å is incident on lithium, obtain the maximum kinetic energy of the photoelectrons and the value of the stopping voltage V_0. 5

(1eV=1.6X10^{-19} J, h=6.63X10^{-34} J.S.)

3.

a) Consider a particle in a one-dimensional potential well $-a/2 \le x \le a/2$ where V = 0 inside and V = ∞ outside.

i) Determine the eigen values and eigenfunctions of this particle 10

ii) The dipole moment operation is defined as eX where e is the charge and X is the position operator.

Find the matrix elements of the dipole moment of the particle. 10

4.

a) A hydrogen atom of which the wavefunction at t= 0 is the following superposition of energy eigenfunctions $\psi_{nlm}(r)$

$$\psi(r,t=0)=\frac{1}{\sqrt{14}}[2\psi_{100}(r)-3\psi_{200}(r)+\psi_{322}(r)]$$

(i) The parity operator Π is defined so that

$$\Pi\psi(r)=(-1)^{l}\psi(r)$$

Is the wave function an eigenfunction of the parity operator ? 5

b) What is the probability of finding the system in the ground state? In state (2,0,0)? In state (3,2,2)? In other state ? 5

b) An electron in the ground state of atomic hydrogen has wave function

$$\psi_{100}(r,\theta,\phi)=\frac{1}{\sqrt{\pi a_0^3}}\exp(-r/a_0)$$

Calculate the expectation values of r, the potential energy, and the kinetic energy. 10

5.

$$S = \frac{\hbar}{2}\sigma$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

a) A Stern-Gerlach apparatus prepares a pure state of spin half particles in the $|X,\uparrow\rangle$ (i.e., a measurement of spin up in the x direction) Subsequently a measurement of the spin in z direction is carried out. What is the probability obtaining a value of spin up in this direction ? 5

b)Consider two spin ½ particles in states $|\tfrac{1}{2},\pm\rangle_1$ and $|\tfrac{1}{2},\pm\rangle_2$ respectively. Show that the product states (4 in number) are eigenfunctions of $S_z = S_{1z} + S_{2z}$ but not of $S^2 = (S_1 + S_2)^2$

What will be the simultaneous eigenstates of S^2 and S_z? 15