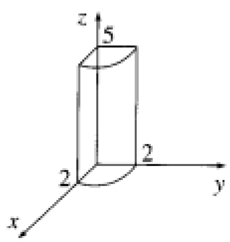
ISI – Bangalore Center – B Math - Physics III – Mid Term Exam
Date: 11 September 2019. Duration of Exam: 3 hours
Total marks: 70
Answer ALL Questions

Q 1. [Total Marks: 5+5+5=15]

a.) If we calculate the total flux $\iint \vec{v}.d\vec{a}$ of a vector field

$\vec{v} = s(2+\sin^2\phi)\hat{s} + s(\sin\phi\cos\phi)\hat{\phi} + (3z)\hat{z}$

over the surface of the quarter –cylinder (radius 2, height 5 units) as shown below.



will the total flux be negative, zero, positive or can it not be decided with the given information?

b.) A vector field is defined in all space except possibly at the origin as
$\vec{F} = \frac{1}{r^2}e^{-r/R}\hat{r}$ where R is a parameter with dimension of length.
Determine if this vector field can be written as a gradient of a scalar field except perhaps at the origin. Justify your answer.

c.) If the field in part b.) represents an electric field, calculate the charge contained in a sphere of radius R.

Q2. [Total Marks: 6+4=10]

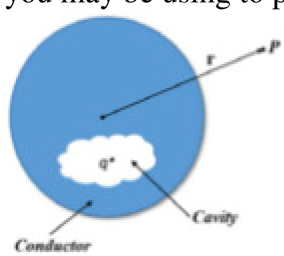
a.) Consider a sphere of radius R with its center at the origin containing charge of uniform volume charge density ρ. A spherical cavity of radius b is then cut out from this sphere. The center of this hollow cavity is $\vec{a}$. Assume that $a + b < R$. Show that the electric field inside the cavity is constant and find its magnitude and direction.

b.) If the total electrostatic energy of the above configuration is considered as a volume integral over all space, calculate the contribution to this energy from the hollow cavity if the cavity is also centered at the origin.

Q3. **[Total Marks: 6+9= 15]**

a.) Suppose that you are given an electrostatic configuration of a fixed non uniform distribution of surface charges with surface charge density $\sigma(p)$ where p is a point on the surface. Discuss quantitatively the continuity properties of the electric field just above and just below any point on the surface.

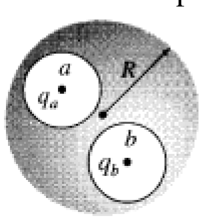
b.) A charge q is kept inside the cavity of an otherwise unirom solid and uncharged metallic sphere without the charge touching the metallic surface. Show that the induced charge on the outer surface of the sphere is independent of the shape of this cavity or the location of the charge inside. Determine its value. Your argument must be quantitative and based on laws of electrostatics. State clearly and accurately what general results of electrostatics you may be using to prove the above.



Q4. [Total Marks: 15]

[If you have answered Q3b above, then you can use the arguments used there without repeating them here. If you have not answered Q 3b, then you will have to provide some justifications for your steps.]

Two spherical cavities of radii a and b are hollowed out from the interior of an uncharged conducting sphere of radius R. Charges are placed at the center of each cavity as shown in the picture:



a.) Find the surface charges $\sigma_a, \sigma_b, \sigma_R$.
b.) What is the field outside the conductor?
c.) What is the field within each cavity?
d.) What is the force on each of the charges?
e.) Which of these answers will change if a third charge is brought near the conductor?

Q5. Total Marks: 2+3+6+4=15

An infinitely long non magnetic cylindrical conductor with inner radius a and outer radius b carries a steady current *I.* Assume that the current density in the conductor is uniform. Choose the axis of the cylinders as z axis, positive z axis as the direction of the current and use cylindrical coordinates.

a.) What is the expression for current density vector $\vec{j}(s,\theta,z)$?
b.) Prove using Biot-Savart law, the component of the magnetic field along the z direction is zero.
c.) Assuming the magnetic field is only in the $\hat{\theta}$ direction calculate the $\vec{B}$ everywhere.
d.) Justify the statement that the magnetic field is only in the $\hat{\theta}$ direction.

Information you may need: Divergenece in cylindrical coordinates and line element in spherical coordinates.

$$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}.$$

$$d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}.$$