Physics III
ISI B.Math
Midterm Exam : September 7 ,2016

Total Marks: 70
Time : 3 hours
Answer all questions

1. (**Marks: 2 + 5 + 7 = 14**)

(a) Write down the volume charge density ρ for the following charge configuration: A positive charge $+q$ located at $\mathbf{r} = \mathbf{a}$ and a negative charge $-q$ located at $\mathbf{r} = -\mathbf{a}$, where $\mathbf{a}$ is a constant vector.

(b) Suppose the electric field in some region is found to be $\mathbf{E} = kr^3\hat{\mathbf{r}}$ in spherical coordinates, where k is a constant. (i) Find the total charge contained in a sphere of radius R centred at the origin. (ii) Find the volume charge density $\rho(r)$.

(c) Find the potential inside and outside a uniformly charged solid sphere whose radius is R and whose total charge is q. Use infinity as your reference point. Sketch $V(r)$ (Hint: You may find it convenient to first compute the electric field)

2. (**Marks: 2 + 4 + 8 = 14**)

(a) It is given that $\nabla \cdot \mathbf{E} = C(\mathbf{r})$ and $\nabla \times \mathbf{E} = \mathbf{0}$. Is it possible to uniquely determine the electric field $\mathbf{E}$ from this data ? Explain.

(b) A point charge q is located at the origin. Show that $\nabla \times \mathbf{E} = 0$ for the electric field $\mathbf{E}$ of this charge. From this result show that this result is true for *any* static charge distribution.

(c) A sphere of radius R carries a charge density $\rho(r) = kr$ (where k is a constant). Find the energy of the configuration.

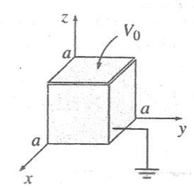
3. (**Marks: 3 + 4 + 7 = 14**)

(a) Use the uniqueness theorem to show that the potential is constant inside an enclosure completely surrounded by conducting material, provided there is no charge within the enclosure.

(b) An infinite grounded plane conductor lies on the $x - y$ plane. Two point charges of magnitude $-2q$ and $+q$ are placed on the z axis at $(0, 0, d)$ and $(0, 0, 3d)$ respectively. Find the force on the charge $+q$.

(c) Now remove the charge $+q$ in the set up of (b) and with this modified configuration find the induced surface charge density on the conducting plane and hence the total charge induced on the plane.

4. (**Marks: 10 + 4 = 14**)



A cubical box (sides of length a) consists of five metal plates, which are welded together and grounded as shown in the figure. The top is made of a separate sheet of metal, insulated from the others and held at a constant potential V_0. (i) Use the method of separation of variables to find the potential inside the box. (ii) Find the surface charge density $\sigma(x, y)$ on the plate at $z = 0$.

5. (**Marks: 5 + 5 + 4 = 14**)

(a) A dielectric cube of side a, centered at the origin, carries a "frozen-in" polarization $\mathbf{P} = k\mathbf{r}$ where k is a constant. Find all the bound charges and show that they add up to 0. Find the value of $\nabla \times \mathbf{D}$ for this configuration, where $\mathbf{D}$ is the electric displacement vector.

(b) A long straight wire, carrying uniform line charge λ per unit length is surrounded by rubber insulation out to a radius a in the form of an infinite cylinder of radius a whose axis is along the line charge. Find the electric displacement $\mathbf{D}$ inside and outside the insulation. Is it possible to determine the corresponding electric field everywhere from this data ? Explain.

(c) Four particles , one of charge q, one of charge $3q$ and two of charge $-2q$, are located at $(0, 0, a), (0, 0, -a), (0, a, 0)$ and $(0, -a, 0)$ respectively. Find an approximate formula for the potential, valid at points far from the origin. (express your answer in spherical coordinates)