Physics III
ISI B.Math
Midterm Exam : September 7 ,2016

Total Marks: 70
Time : 3 hours
Answer all questions

1. (**Marks:** $5 \times 2 = 10$)

(a) An electric dipole consists of a charge $+q$ at $(0,0,d)$ and a charge $-q$ at $(0,0,-d)$. $\mathbf{E}$ is the electric field due to the dipole. Which of the following statements is false ?

(i) $\nabla \times \mathbf{E} = 0$
(ii) $\nabla \cdot \mathbf{E} = 0$
(iii) the monopole moment of the charge distribution is zero.
(iv) The volume charge density $\rho(\mathbf{r}) \neq 0$

(b) A hollow conductor of arbitrary shape has a net charge Q on it. Which of the following statements is false ?

(i) The electrostatic potential V at the surface of the conductor is a constant $V = C$
(ii) The electrostatic potential V inside the hollow conductor is a constant $V = C$ (the same value as in (a))
(iii) the electric field $\mathbf{E}$ at the surface of the conductor is zero.
(iv) At the surface of the conductor $\mathbf{E} = -\nabla V$

(c) A point charge $+q$ is placed at $(0,0,d)$ at a distance d above an infinite grounded conducting plane ($x-y$ plane). The energy of the configuration is

(i) $\frac{1}{4\pi\epsilon_0}\frac{q^2}{4d}$
(ii)-$\frac{1}{4\pi\epsilon_0}\frac{q^2}{4d}$
(iii)-$\frac{1}{4\pi\epsilon_0}\frac{q^2}{2d}$
(iv) $\frac{1}{4\pi\epsilon_0}\frac{q^2}{2d}$

(d) Consider a slab of dielectric with a bound charge ρ_b and a free charge ρ_f on it. If $\mathbf{D}$ is the displacement vector and $\mathbf{E}$ the electric field, which of the following statements is false in general ?

(i) $\nabla \times \mathbf{E} = 0$
(ii) $\nabla \times \mathbf{D} = 0$
(iii) $\nabla \cdot \mathbf{E} = \frac{\rho_b + \rho_f}{\epsilon_0}$
(iv) $\nabla \cdot \mathbf{D} = \rho_b$

(e) Consider a stationary configuration of source charges. A test charge Q is moved from point $\mathbf{a}$ to $\mathbf{b}$ in the field of the source charges. Which of the following statements is true ?
(i) The work done by the field on Q to move it from $\mathbf{a}$ to $\mathbf{b}$ is always independent of the path.
(ii) The work done by the field on Q to move it from $\mathbf{a}$ to $\mathbf{b}$ and back to $\mathbf{a}$ is not equal to zero

(iii) The work done by the field on Q to move it from $\mathbf{a}$ to $\mathbf{b}$ is independent of path for only specific charge configurations.
(iv)The work done by the field on Q to move it from $\mathbf{a}$ to $\mathbf{b}$ always depends on the path

2. (**Marks: 4 + 4 + 7**)

(a) Let $\mathbf{s}$ be the separation vector from a fixed point (x', y', z') to the point (x, y, z) and let s be its length. Show that $\nabla\left(\frac{1}{s}\right) = -\left(\frac{\hat{\mathbf{s}}}{s^2}\right)$.
(b) If it was discovered that the electrostatic force between two charges was proportional to $\frac{1}{r^3}$ rather than $\frac{1}{r^2}$, would it still be possible to associate a scalar potential function with such a force? Will Gauss's law continue to hold ? Explain.
(c) A hollow spherical shell carries charge density $\rho = \frac{k}{r^2}$ in the region $a \leq r \leq b$. Find electric field $\mathbf{E}$ and plot $|\mathbf{E}|$ as a function of r.

3. (**Marks: 8 + 4 + 3**)

A uniform of charge λ per unit length is placed on an infinite straight wire a distance d above a grounded conducting plane (let us say that the wire runs parallel to the x- axis and the conducting plane is the $x - y$ plane

(a) Find the potential in the region above the plane.
(b) Find the charge density σ induced on the conducting plane
(c) From (b) Show that the total charge per unit length (in the y direction) on the conducting surface is $-\lambda$

4. (**Marks: 5 + 10**)

(a) Given an arbitrary charge distribution $\rho(\mathbf{r})$, show that the dipole moment (in the sense of a multipole expansion) of the charge distribution is independent of the origin only if the total charge, i.e, the monopole moment vanishes.
(b)An infinitely long metal pipe of square cross-section of side a is grounded, but one end, at $(x = 0)$ is maintained at a constant potential V_0.
Use the method of separation of variables to show that the potential inside pipe is given by

$$V(x, y, z) = \frac{16V_0}{\pi^2} \sum_{n,m=1,3,5\cdots}^{\infty} \frac{1}{nm} e^{-\frac{\pi}{a}\sqrt{m^2+n^2}x} \sin\frac{n\pi y}{a} \sin\frac{m\pi z}{a}$$

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5. Marks (9 + 6)

A thick spherical shell(inner radius a, outer radius b) is made of dielectric material with a "frozen-in" polarization

$$P(\mathbf{r}) = \frac{k}{r}\hat{\mathbf{r}}$$

where k is a constant and r the radius from the centre. (There is no free charge in the problem).
(a) Locate all the bound charge, and use Gauss's law to calculate the electric field $\mathbf{E}$ it produces.
(b) Use Gauss's law for the displacement field $\mathbf{D}$ and calculate $\mathbf{E}$ from it using the given polarization. Do the results of (a)and (b) agree? What is the value of the curl of $\mathbf{E}$?