Physics III
ISI B.Math
Mid Semestral Exam : September 20, 2012

Total Marks: 80
Answer any four questions.

1. Marks (4 + 9 + 3 + 4)

(a) Find a unit vector normal to the surface $xy^2 + xz = 2$ at the point $(1, -1, 1)$

(b) Show that $\mathbf{F} = yz\hat{\mathbf{x}} + zx\hat{\mathbf{y}} + zx\hat{\mathbf{z}}$ can be written as both a gradient of a scalar field and as a curl of a vector field . Find the corresponding scalar and vector fields. Are the scalar and vector fields that you found unique? Explain your answer. Can **F** represent a real electrostatic field?

(c) What is the volume charge density $\rho(\mathbf{r})$ of an electric dipole, consisting of a point charge $-q$ at the origin and a point charge $+q$ at **a**?

(d) Evaluate the integral $\int_{\mathcal{V}} e^{-r}\left(\nabla\cdot\frac{\hat{\mathbf{r}}}{r^2}\right)d\tau$ where $\mathcal{V}$ is a sphere of radius R centred at the origin.

2. Marks (5 + 5 + 10)

(a) A pyramid has a square base of side a, and four faces which are equilateral triangles. A charge Q is placed on the centre of the base of the pyramid. What is the net flux of the electric field emerging from one of the triangular faces of the pyramid?

(b) A charge distribution gives rise to a radial electric field $\mathbf{E} = \frac{a}{r^2}e^{-\frac{r}{b}}\hat{\mathbf{r}}$. What is the total charge giving rise to this electric field?

(c) A sphere of radius R carries a charge density $\rho(r) = kr$(where k is a constant). Find the energy of the configuration.

3. Marks (7 + 3 + 10)

(a) A hollow metallic sphere is initially uncharged. Now imagine that a positive point charge $+q$ is placed somewhere inside the sphere (not at the centre) without touching the walls. Make a qualitative sketch of the relative concentration of induced charges on the inner and outer surfaces of the sphere using the symbols + and -. Sketch the electric field lines inside and outside the sphere. Give a brief justification for your sketch. Explain how the charge distribution on the outer sphere will change if the point charge is moved around inside the cavity.

(b) Use the uniqueness theorem for Laplace's equation and the known properties of conductors to show that the electric field in the empty space inside a hollow conductor is zero.

(c) A point charge q is held a distance d above a grounded infinite conducting plane. If the conducting plane lies in the $x - y$ plane, find the induced surface charge distribution $\sigma(x, y)$ on the plane and the energy of the configuration.

4. Marks (12 + 6 + 2)

A thick spherical shell(inner radius a, outer radius b) is made of dielectric material with a "frozen-in" polarization

$$P(\mathbf{r}) = \frac{k}{r}\hat{\mathbf{r}}$$

where k is a constant and r the radius from the centre. (There is no free charge in the problem).
(a) Locate all the bound charge, and use Gauss's law to calculate the electric field $\mathbf{E}$ it produces.

(b) Use Gauss's law for the displacement field $\mathbf{D}$ and calculate $\mathbf{E}$ from it using the given polarization. Do the results of (a) and (b) agree?

(c) What is the value of the curl of $\mathbf{E}$?

5. Marks (13 + 7)

(a)Two infinite grounded metal plates lie parallel to the xz plane, one at $y = 0$ and the other at $y = a$. The left end, at $x = 0$ is closed off with an infinite strip insulated from the other two and maintained at a constant potential V_0. Find the potential inside the slot

(b) Two point charges $3q$ and $-q$ are separated by a distance a in two different configurations (i) $-q$ at (0,0,0) and $+3q$ at $(0, 0, a)$ (ii) $-q$ at $(0, 0, -a)$ $3q$ at $(0, 0, 0)$. Find the monopole moment and the dipole moment for each configuration. Find the approximate potential (in spherical coordinates) at large r (include both the monopole and dipole contribution).

Information you may need to use:

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r \sin\theta}\frac{\partial A_\phi}{\partial \phi}$$