

Physics III
ISI B.Math
Mid Semestral Exam : September 13, 2005

Total Marks: 100

Answer all questions.

1. (i) The vector potential \mathbf{A} of a magnetic dipole of dipole moment \mathbf{m} is given by

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \left(\frac{\mathbf{m} \times \mathbf{r}}{r^3} \right)$$

Show that the magnetic induction $\mathbf{B} = \nabla \times \mathbf{A}$ is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{m}) - \mathbf{m}}{r^3}$$

(ii) In a (nonrotating) isolated mass such as a star, the condition for equilibrium is

$$\nabla P + \rho \nabla \phi = 0$$

Here P is the total pressure, ρ the density, and ϕ the gravitational potential. Show that at any given point the normals to the surfaces of constant pressure and constant gravitational potential are parallel.

2. A static charge distribution produces a radial electric field

$$\mathbf{E} = A \frac{e^{-br}}{r} \hat{\mathbf{r}}$$

where A and b are constants.

(a) What is the charge density? Sketch it.

(b) What is the total charge Q ?

3. (a) Justify **Earnshaw's Theorem**: *A charged particle cannot be held in a stable equilibrium by electrostatic forces alone.*

(b) Two uniform infinite sheets of electric charge densities $+\sigma$ and $-\sigma$ intersect at right angles. Find the magnitude and direction of the electric field everywhere and sketch the lines of \mathbf{E} .

(c) Find the total energy of a uniformly charged spherical shell of total charge q and radius R .

4. Charges $+q$, $-q$ lie at the points $(x, y, z) = (a, 0, a), (-a, 0, a)$ above a grounded conducting plane at $z = 0$. Find
- The total force on the charge $+q$.
 - The work done against the electrostatic forces in assembling this system of charges.
 - The surface charge density at the point $(a, 0, 0)$.
5. (a) Find the potential $V(r, \theta)$ and the corresponding electric field for an electric dipole of dipole moment \mathbf{p} . Choose the dipole to be at the origin with its dipole moment pointing in the z direction.
- (b) Find the bound charges and the electric field produced by a uniformly polarized sphere (with polarization \mathbf{P}) of radius R . [*Hint: model the uniformly polarized sphere as two uniformly charged spheres – one positively charged and one negatively charged superposed with a slight displacement of their centres. Justify this model in a few lines before you proceed.*]
- (c) A metal sphere of radius a carries a charge Q . It is surrounded, out to radius b by a linear dielectric material of permittivity ϵ . Find the potential at the centre (relative to infinity).
6. (a) Show that kinetic energy of a charged particle is conserved when it is moving in a pure magnetic field.

- (b) Find the magnetic field at point P for the steady current configuration shown in the figure
- (c) Is Ampere's law consistent with the general rule that divergence of a curl is always zero? Show that Ampere's Law cannot be valid outside magnetostatics.
- (d) Show that the magnetic vector potential \mathbf{A} corresponding to a magnetic field \mathbf{B} can always be made to satisfy the following equation by exploiting the freedom in defining the vector potential appropriately.

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

where \mathbf{J} is a steady current density.