

ISI – Bangalore Center – B Math - Physics III –Back Exam
Date: January 2015. Duration of Exam: 3 hours
Total marks: 50

Answer Q1, Q2, Q3 and either Q4 OR Q5.

Q1. [10 Marks]

- a.) A infinite slab of thickness $2d$, carries a uniform volume charge density ρ . Let the center of the slab correspond to $z=0$. Determine the electric field everywhere. [Hint: use symmetry of the problem and an appropriate Gauss surface]
- b.) Calculate the electrostatic energy contained in a cube of side $2a$ ($a > d$) which is positioned symmetrically around the slab.

Q2. [10 Marks]

What is the mathematical condition for current density $\vec{j}(x, y, z, t)$ for the Ampere's law to hold?
Express the same condition in terms of the charge density $\rho(x, y, z, t)$.

Write the expression for the charge density for a point charge moving with velocity v (assume that the charge is at the origin at $t=0$.) and explain why Ampere's law is not valid in this case.

State the Biot-Savart law of magnetic field produced by a current I . Using the Biot-Savart law, show that the magnitude of the magnetic field at the center of a circular ring of radius R carrying current I is given by $\frac{\mu_0 I}{2R}$. What is the direction of the magnetic field?

Q3. [10 Marks]

Two infinite parallel wires separated by a distance d are carrying equal and opposite current I which is changing at the rate of $\frac{dI}{dt}$. A square loop of side d is kept with one side parallel to one of the wires at a distance d from that wire. Assume that the plane of the square loop is same as the plane of the wires.

Show that the emf induced is given by $-\frac{\mu_0 d}{2\pi} \ln\left(\frac{4}{3}\right) \frac{dI}{dt}$.

What is the direction of the current in the loop (clockwise or anticlockwise)?

DO EITHER Q4 OR Q5.

Q4. [20 Marks]

4a.) Show that for a point charge, the average of the electric potential due to the charge over any sphere not containing the charge is equal to the value of the potential at the centre of the sphere.

4b.) Use the result in part 4a to prove that the electric field inside a closed volume is uniquely determined if the electrostatic potential is specified everywhere on the surface.

Q5. [20 Marks]

5a.) State the Maxwell Equation in matter characterized by ϵ, μ where there is no free charge or free current.

5b.) Derive the boundary conditions satisfied by the perpendicular and parallel components of electric and magnetic fields at the interface of two dielectric media.