# End Semester Examination

Physics III,
B. Math., $3^{rd}$ year,
September - December 2021.

Instructor: Prabuddha Chakraborty (pcphysics@gmail.com)

December $29^{th}$, 2021, Morning Session.
Duration: 3 hours.
Total points: 90.

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Grades will be awarded not only based on what final answer you get, but also on the intermediate steps.

1. **(Total: 15 points)** A charge $q$ is located inside a hollow spherical shell of radius $R$ and negligible thickness, at a distance $a$ from the center of shell ($a < R$). The shell is conducting and grounded. Find
   (a) The potential everywhere in polar co-ordinates. Take the center of the spherical shell as the origin.[8]
   (b) The surface charge density on the conducting shell. [5]
   (c) The force on the charge $q$ (both magnitude and direction). [2]
2. **(Total: 15 points)**
   (a) Two point electric dipoles in vacuum, $\vec{p_1}$ and $\vec{p_2}$ are constrained to be co-planar, i.e., the orientations of the two dipoles are free to rotate on the same plane. Also their orientations are co-planar with the line joining them. Let $r$ denote the distance between them (they are not allowed to accelerate along $\pm\hat{e}_{\vec{r}}$). If they make the angles $\theta_1$ and $\theta_2$ respectively with this line, find the relationship between $\theta_1$ and $\theta_2$ in mechanical equilibrium. [7]
   (b) A uniformly charged ring of radius $a$ carrying total charge $-q$ is placed as a concentric circle with another uniformly charged ring of radius $b$ with total charge $+q$ ($b > a$). Find the electric potential (with terms up to and including $\frac{1}{r^5}$ - dependence) at a distance $r$, $r \gg a, b$. [8]
3. **(Total: 15 points)**

(a) A sphere of radius $R_1$ has a uniform charge density $\rho$ within its volume, except for a small hollow sphere of radius $R_2$, whose center is located at a distance $a$ from the center of the $R_1$-sphere ($R_2 < a$, $R_2 + a < R_1$). Find the electric potential at the center of the hollow sphere. [5]

(b) A parallel-plate capacitor is charged to potential difference $\Delta\phi$ and then disconnected from the charging circuit. Now the distance between the two plates is slowly changed from $d_i$ to $d_f$, with $d_f > d_i$. How much work (including sign) needs to be done by an external agent to do so? Assume no work is wasted, all charges are at rest at the beginning and end of the process, and the plates are circular with radius $r \gg d_f$. [5]

(c) There are two electrodes (consider them to be equipotential parallel plates at distance $d$) held at electric potential 0 and $\Phi_0 (> 0)$. If an unlimited supply of electrons (mass $m_e$) is supplied to the grounded electrode, they will move towards the other electrode. When steady current is established, what is the value of this steady current? Assume that no external work is being done except the hold the electrodes at fixed potential difference, in spite of a steady current of electrons. [5]

4. **(Total: 15 points)**

(a) There are two co-axial cylindrical conductors, with the inner radius $a$ and outer radius $b$, and the intervening space is filled with a dielectric medium with dielectric constant $\kappa$. The medium also has a conductivity $\sigma$, which helps us to relate the electric field at a given point in space to the local current density through Ohm's law, $\vec{j} = \sigma\vec{E}$. Find the resistance $R$ between the inner and the outer conductor as well as the capacitance $C$ of this system. To find the resistance, you may use the more popular version of Ohm's Law, $\Delta\phi = IR$. You may assume that the cylinders have length $l \gg b$. [5]

(b) Find the potential energy of a point charge $q$ in vacuum which is at a distance $d$ from a semi-infinite dielectric medium of dielectric constant $\kappa_0$. [10]

5. **(Total: 15 points)** Consider three infinitely long, co-planar wires placed parallel to each other. Each carries the same current $I$ in the same direction. The middle wire is placed at equal distance $d$ from the other two.

(a) Find the location of the zeroes of the magnetic field [5]

(b) The middle wire is displaced a short distance $x$ towards one of the wires, remaining parallel to its original orientation throughout the displacement process. Describe its subsequent motion in detail for $x \ll d$. Imagine the others are held in place. [5]

(c) Now the middle wire is displaced perpendicular to the common plane by the same distance $x$, again remaining parallel to its original orientation throughout. Like in the previous part, describe the subsequent motion of the middle wire. The other wires are always held in place. [5]

6. **(Total: 15 points)**

(a) A current $I$ travels down towards the $z = 0$ plane $-\hat{e}_z$ from $z \to \infty$ and then spreads out uniformly and radially along the $z = 0$ plane.

i. Note that both reflection about the $y - z$ plane as well as a counter-clockwise rotation by $\pi$ about the $z$-axis leave the current density invariant. Use the properties of the transformations of $\vec{B}$ under these two symmetry transformations as well as any other required symmetry arguments to show that $\vec{B}(\vec{r}) = B_\phi(\rho, z)\hat{e}_\phi$, $\forall \vec{r}$ in $\mathbb{R}^3$ [4].

ii. Find $\vec{B}$ everywhere. [4]

iii. Demonstrate that at $z = 0$, the solution you have obtained satisfies the matching conditions for $\vec{B}(\vec{r})$. [2]

(b) In the Coulomb gauge, $\vec{\nabla} \cdot \vec{A} = 0$, current density in a given Cartesian direction, say $\hat{e}_x$, produces a vector potential in the same Cartesian direction only. This is generally not true for mutually orthogonal but non-Cartesian co-ordinates. However, for current densities which are azimuthally symmetric, show that a current density that can be written as $j(r, \theta)\hat{e}_\phi$ in spherical co-ordinates produces a vector potential only in the $\hat{e}_\phi$ direction. [5]