Physics III
ISI B.Math
Final Exam : November 11 ,2016

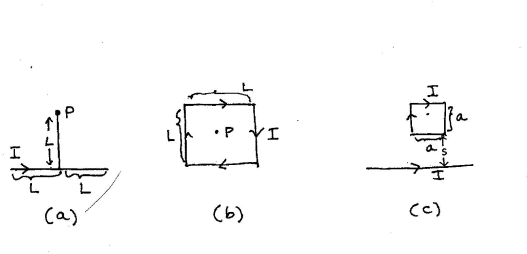
Total Marks: 50
Time : 3 hours
Answer all questions

1. (**Marks: 4 + 6 = 10**)

A sphere of radius R carries a charge density $\rho(r) = kr$.

(a) Find the electric field $\mathbf{E}(\mathbf{r})$ for this charge distribution.

(b) Find the energy of this configuration .



2. (**Marks: 5 + 2 + 3 = 10**)

(a) Find the magnetic field at the point P (see fig (a)) at a distance L from a wire segment of length $2L$ carrying a current I

(b) Find the magnetic field at the point P (see fig (b)) at the centre of a square loop of wire of side L carrying a current I.

(c) Find the force on a square loop of wire of side a (see fig (c)) carrying a current I at a distance s from a long wire carrying current I.

3. (**Marks: 2 + 5 + 2 + 1 = 10**)

A very long air core solenoid of radius b has n turns per unit length and carries a current $i = i_0 \sin \omega t$

(a) Write an expression for the magnetic field $\mathbf{B}$ inside the solenoid as a function of time.

(b) Find the electric field $\mathbf{E}$ inside and outside the solenoid as a function of time .

(c) Make a rough graph of $|\mathbf{E}|$ vs r (the distance from the axis of the solenoid) at time $t = \frac{2\pi}{\omega}$.

(d) Make a rough sketch of the electric field lines at $t = \frac{2\pi}{\omega}$.

4. (**Marks: 2 + 2 + 2 + 4 = 10**)

(a) Write down the full set of Maxwell's equations for the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ in the presence of a charge density ρ and current density $\mathbf{J}$.

(b) Explain how the equations in (a) will be modified if it was discovered that magnetic monopoles exist in nature

(c) Identify the "displacement current" term added by Maxwell. Explain the necessity of adding this term.

(d) Starting from source free Maxwell's equations in vacuum, show that each component of the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ obeys a wave equation. Find the velocity of the wave in terms of ϵ_0 and μ_0.(Note that :$\nabla \times \nabla \times \mathbf{A} = -\nabla^2 \mathbf{A} + \nabla(\nabla \cdot \mathbf{A})$)

5. (**Marks: 1 + 1 + 4 + 4 = 10**)

A plane wave solution of source free Maxwell's equations is given by $\mathbf{E} = E_0 \cos(kz - \omega t)\hat{\mathbf{x}}$ where E_0 is the real amplitude, k the wave number and ω the angular frequency of the wave.

(a) In which direction is the wave propagating ?

(b) In which direction is the wave polarized ?

(c) If the corresponding solution for the magnetic field is given by $\mathbf{B} = B_0 \cos(kz - \omega t)\hat{\mathbf{n}}$, find B_0 in terms of E_0 from Maxwell's equations. Also find the direction of the unit vector $\hat{\mathbf{n}}$

(d) Find the energy density u and Poynting vector $\mathbf{S}$ for the above electromagnetic wave in terms of E_0 and verify that $|\mathbf{S}| = cu$ where c is the velocity of light. What does the Poynting vector physically represent ?