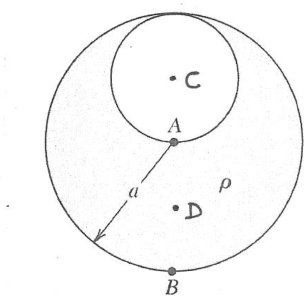
Physics III
ISI B.Math
Final Exam : November 2 ,2015

Total Marks: 50
Time : 3 hours
Answer all questions

1. (**Marks: 6 + 2**)

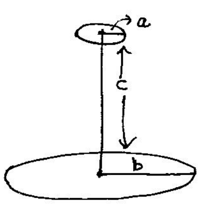


(a) A solid non-conducting sphere of radius a is filled with positive charge of uniform charge density ρ. Then a smaller sphere of radius $\frac{a}{2}$ is carved out as shown in the figure and left empty. What are the direction and magnitude of the electric field $\mathbf{E}$ at A? At B?

(b) Find $\nabla \cdot \mathbf{E}$ and $\nabla \times \mathbf{E}$ at the points C and D , where C and D are points in the interior of the empty region and the charge filled region respectively.

2. (**Marks: 5 + 5**)

(a) A particle with charge q is travelling with a velocity $\mathbf{v}$ parallel to a wire with a uniform linear charge distribution λ per unit length. The wire also carries a current I in the same direction as the velocity of the particle. What must be the velocity of the the particle in order to travel in a straight line parallel to the wire a distance r away ?



(b) Two single turn circular loops are mounted as shown in the above figure. Find the mutual inductance between the coils assuming $a \ll b$.

3. (**Marks: 3 + 2 + 3 + 4**)

(a) Write down the full set of Maxwell's equations in differential form. What is the significance of the asymmetry between the equations involving the divergence of the electric and magnetic fields?

(b) Explain how the “displacement current” term added to Ampere’s law by Maxwell removes the inconsistency in Ampere’s Law for non-static magnetic fields.

(c) Show that a magnetic field $\mathbf{B}$ and electric field $\mathbf{E}$ that is a solution to Maxwell’s equations can always be written as

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$$

(d) Show that , for Maxwell’s equations in vacuum, each Cartesian component of $\mathbf{E}$ and $\mathbf{B}$ satisfies the 3-D wave equation

$$\nabla^2 f = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

with $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$. Note that (Note that :$\nabla \times \nabla \times \mathbf{A} = -\nabla^2 \mathbf{A} + \nabla(\nabla \cdot \mathbf{A})$)

4. (**Marks: 1 + 2 + 3 + 4**)

Consider the following electric and magnetic fields whose real parts represent monochromatic plane waves propagating with velocity $c = \frac{\omega}{k}$. $\tilde{\mathbf{E_0}}$ and $\tilde{\mathbf{B_0}}$ are complex amplitudes.

$$\tilde{\mathbf{E}}(x,t) = \tilde{\mathbf{E_0}} e^{i(kx-\omega t)}$$

$$\tilde{\mathbf{B}}(x,t) = \tilde{\mathbf{B_0}} e^{i(kx-\omega t)}$$

(a) What is the direction of propagation of the waves and what is the frequency ?

(b) If the waves satisfy source free Maxwell’s equations in vacuum , show that the waves are transverse

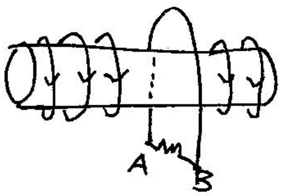
(c) If the waves satisfy source free Maxwell’s equations in vacuum, show that

$$\tilde{\mathbf{B_0}} = \frac{1}{c}(\hat{\mathbf{x}} \times \tilde{\mathbf{E_0}})$$

Hence, if the direction of polarization of the electric field is $\hat{\mathbf{z}}$, what is the direction of polarization of the magnetic field?

(d) Find the Poynting vector for such a wave in terms of E_0, the real amplitude for the electric field . Calculate the time average of the Poynting vector over one time period of the wave. What does the Poynting vector represent physically ?

5. (**Marks: 4 + 4 + 2**)



A long solenoid of radius a, carrying n turns per unit length, is looped by a wire with resistance R as shown in the above figure.

(a) If the current in the solenoid is increasing at a constant rate ($\frac{dI}{dt} = k$) what current flows in the loop and which way (from A to B or from B to A) does it pass through the resistor ?

(b) If the current I in the solenoid is constant but the solenoid is pulled out of the loop and reinserted in the opposite direction, what total charge passes through the resistor ?

(c) It is claimed that the energy "stored in a magnetic field " $\mathbf{B}$ is given by $\int B^2 d\tau$ where the integral represents a volume integral over all space. Explain in what sense a magnetic field can store energy though magnetic forces do no work .