ISI – Bangalore Center – B Math - Physics III – End Semestral Exam
Date: 12 November 2014. Duration of Exam: 3 hours
Total marks: 50

Q 1. [Total Marks: 10]

For Q1, ONLY FINAL ANSWERS NEED TO BE GIVEN NEATLY . Calculations need not be shown.

1a.) What is the dipole moment vector of the following electrostatic systems?

i.) uniformly charged sphere of total charge Q whose center is at the origin.
ii.) uniformly charged sphere of total charge Q whose center is at a point (0,0,z).

1b.) Write the specific Maxwell Equation (in matter) which leads to the following boundary condition is $B_1^{\perp} = B_2^{\perp}$ across the surface separating two media.

c.) Let $\vec{E}_1 = A\cos(kz - wt)\vec{i}$ and $\vec{E}_2 = A\cos(kz - wt + \delta)\vec{j}$ be two electric fields satisfying Maxwell equation in vacuum propagating in z direction .For what values of δ will $\vec{E}_1 + \vec{E}_2$ represent a circularly polarized light? (All quantities in $\vec{E}_1, \vec{E}_2$ real).

d.) Let $\vec{E}(\mathrm{x,y,z,t})$ and $\vec{B}(x,y,z,t)$ be the fields with charge and current density ρ and j respectively. Let $\vec{E}'(\mathrm{x,y,z,t})$ and $\vec{B}'(x,y,z,t)$ be the fields wihen signs of all charges are reversed. How are $\vec{E}'(\mathrm{x,y,z,t})$, $\vec{B}'(x,y,z,t)$ related to $\vec{E}(\mathrm{x,y,z,t})$, $\vec{B}(x,y,z,t)$?

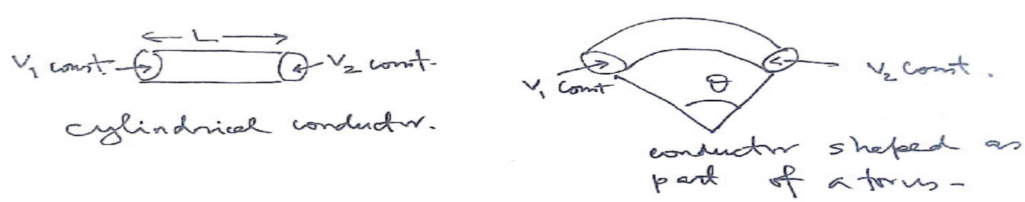
e.) Write the equation that represents the law of local conservation of electric charges.

Q 2. [Total Marks: 4+4+2=10]

2a.) Consider a current carrying conductor of conductivity σ . Assume that there is a steady current I flowing through the conductor. Show that in this case, the electric field inside the conductor can be represented by $\vec{E} = -\vec{\nabla} V$, with $\nabla^2 V = 0$.

2b.) Now suppose that the conductor is cylindrical with an arbitrary cross section and that the two ends of the conductors are kept at constant but different potential. Suppose the potential difference is $V_1 - V_2 = \phi$ and the length of the conductor is L as shown in the picture. Find the electric field inside the conductor and show that it is constant.

2c.) How will your answer for the electric field change if the conductor is part of a torus as shown in the picture? What will be its magnitude and direction? [Make an intelligent guess based on the answer of part 2b. No calculation necessary]



Q 3. [Total Marks: 4+4+2=10]

A rectangle of a perfectly conducting wire (that is ignore any resistance) with sides a and b (b>>a) has mass *M* and self inductance *L* is moving with velocity v_0 in its plane along the direction of the side b. The loop enters a region of uniform magnetic field *B* which is perpendicular to the plane of the wire …

3a.) Complete the following equations:

$M\frac{dv}{dt} = ...$, and $L\frac{dI}{dt} = ...$, where *v* is the instantaneous speed of the loop.

3b.) Show by solving the above equations that displacement of the wire inside the magnetic file is given by

$s = \frac{v_0}{w}\sin wt$, and determine the frequency *w*.

3c.) Explain how the total energy of the system is conserved as the speed of the loop changes with time.

Q 4. [Total Marks: 10]

Suppose that the electric field of an electromagnetic wave is given by the superposition of two waves

$\vec{E} = E_0(\cos(kz - wt)\vec{i} + \cos(kz + wt)\vec{j})$.

4a.) Starting from the appropriate Maxwell Equation, calculate the corresponding magnetic field.

4b) What is the energy per unit area per unit time (the Poynting vector S) transported by this wave?

4c) Calculate the time average of the Poynting S vector over a cycle T? Briefly interpret the result you get.

4d.) If the electric field was $\vec{E} = E_0(\cos(kz - wt)\vec{i} + \cos(kz - wt)\vec{j})$ instead of what is given above, what would be time average of the Poynting vector? (detailed calculation not needed. You can use standard results for plane waves if needed).

Q 5. [Total Marks: 10]

5a.) Write the Maxwell equations in a homogeneous and isotropic dielectric medium (characterized by ε, μ) containing free charge and free current density ρ_f, j_f .

Show that in the absence of a free charges these imply wave equations for E and B fields inside the dielectric medium . What is the speed of propagation of the wave inside the dielectric?

5b.) An EM plane wave with frequency ω_I propagating in z direction in a dielectric medium 1 and is reflected at the interface of a second dielectric media. Let z=0 be the plane of separation of the two media. Assume that the reflected wave and the transmitted wave are both propagating along the normal and are the polarized in the same direction as the incident ray. Let the frequencies of the reflected and the transmitted wave be ω_R, ω_T respectively.

Write the expressions for the electric field for the incident. reflected and transmitted ray. Write the boundary condition related to the electric field components in the z direction. Using this prove that $\omega_I = \omega_R = \omega_T$