Physics III
ISI B.Math
Final Exam : December 3, 2010

Total Marks: 110
Answer all questions.

1.(i) A square loop of wire (side a) lies on a table, a distance s from a very long straight wire, which carries a current I, as shown in the figure 1.
(a) Find the flux of $\mathbf{B}$ through the loop. (5)
(b) If someone now pulls the loop directly away from the wire, at speed v, what emf is generated ? In which direction (clockwise or counterclockwise) does the current flow?(5)
(c) What emf is generated if the loop is pulled to the right at speed v instead of away?(3)
(ii) A short solenoid (length l and radius a, with n_1 turns per unit length) lies on the axis of a very long solenoid (radius b, n_2 turns per unit length)as shown in figure 2. Current I flows in the short solenoid. What is the flux through the long solenoid ? Find the mutual inductance of the system. (9)

2. a) Write down the full set of Maxwell's equations in differential form. What is the significance of the asymmetry between the equations involving the divergence of the electric and magnetic fields?(5)
b) Explain how the "displacement current" term added to Ampere's law by Maxwell removes the inconsistency in Ampere's Law for non-static magnetic fields.(5)
c) A fat wire of radius a, carries a constant current I, uniformly distributed

over its cross-section. A narrow gap in the wire, of width $w \ll a$, forms a parallel plate capacitor, as shown in the figure. Find the magnetic field in the gap, at a distance $s < a$ from the axis.[Hint: Notice that the only source of the magnetic field is the changing electric field within the gap.] (6)
d) Show that , for Maxwell's equations in vacuum, each Cartesian component of $\mathbf{E}$ and $\mathbf{B}$ satisfies the 3-D wave equation

$$\nabla^2 f = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

with $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ (6)

3. Consider the following electric and magnetic fields whose real parts represent monochromatic plane waves propagating with velocity $c = k/\omega$.

$$\tilde{\mathbf{E}}(x,t) = \tilde{\mathbf{E_0}} e^{i(kx-\omega t)}$$

$$\tilde{\mathbf{B}}(x,t) = \tilde{\mathbf{B_0}} e^{i(kx-\omega t)}$$

(i) What is the direction of propagation of the waves and what is the frequency ?(3)
(ii) If the waves satisfy Maxwell's equations, show that the waves are transverse (6)
(iii) If the waves satisfy Maxwell's equations show that (6)

$$\tilde{\mathbf{B_0}} = \frac{k}{\omega}(\hat{\mathbf{x}} \times \tilde{\mathbf{E_0}})$$

Hence, if the direction of polarization of the Electric field is $\hat{\mathbf{z}}$, what is the direction of polarization of the magnetic field?
iv) Find the time average (over a cycle) of the energy density and the Poynting vector for such a wave. What does the Poynting vector represent physically ?(7)
4.(a) Two spheres, each of radius R and carrying uniform charge densities $+\rho$ and $-\rho$ respectively are placed so that they partially overlap, as shown

in fig.(a). Call the vector from the positive centre to the negative centre **d**. Show that the field in the overlap region is a constant and find its value.[Hint: Use the superposition principle] (10)
(b) A metal sphere of radius R carrying a charge q is surrounded by a thick concentric metal shell (inner radius a, outer radius b). The shell carries no net charge.
i) Find the surface charge density σ at R at a and at b (4)
ii) Find the potential at the centre, using infinity as the reference point. (4)
iii) Now the outer surface is touched to a grounding wire, which lowers its potential to zero (same as infinity). How do your answers to (i) and (ii) change?(4)
5.(a) The figure shows an arrangement used by Dempster to measure the

mass of ions. An ion of mass M and charge $+q$ is produced essentially at rest in source S, a chamber in which gas discharge is taking place. The ion is accelerated by a potential difference V and allowed to enter a magnetic field **B**. In the field it moves in a circle, striking a photographic plate at a distance x from the entry slit and being recorded. Show that the mass M is given by $M = \frac{B^2 q}{8V} x^2$. (11)
(b) A battery supplying constant emf $\mathcal{E}_0$ is connected to a circuit of resistance R and inductance L. Find the current $I(t)$ flowing in the circuit and plot it as a function of time. (11)