Physics III
ISI B.Math
Final Exam : November 25, 2005

Total Marks: 100
Answer any five questions.

1. (i) One of the following two fields is an impossible electrostatic field. Which one? Justify your answer.
(a) $\mathbf{E} = k[xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3xz\hat{\mathbf{z}}]$
(b) $\mathbf{E} = k[y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}]$
(ii) Find the net force that the southern hemisphere of a uniformly charged sphere of total charge Q and radius R exerts on the northern hemisphere.
(iii) What charge density $\rho(r)$ would produce a field

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0}\frac{\mathbf{r}}{r^n}?$$

Discuss in particular the case $n = 3$.

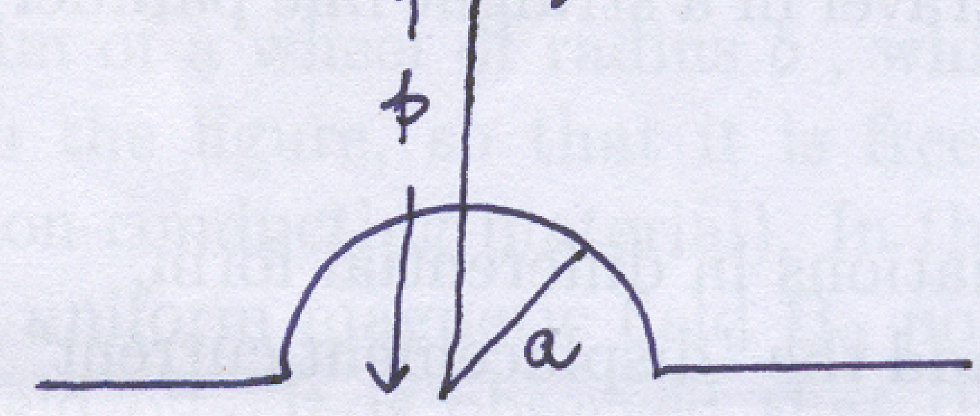


Fig. 2a

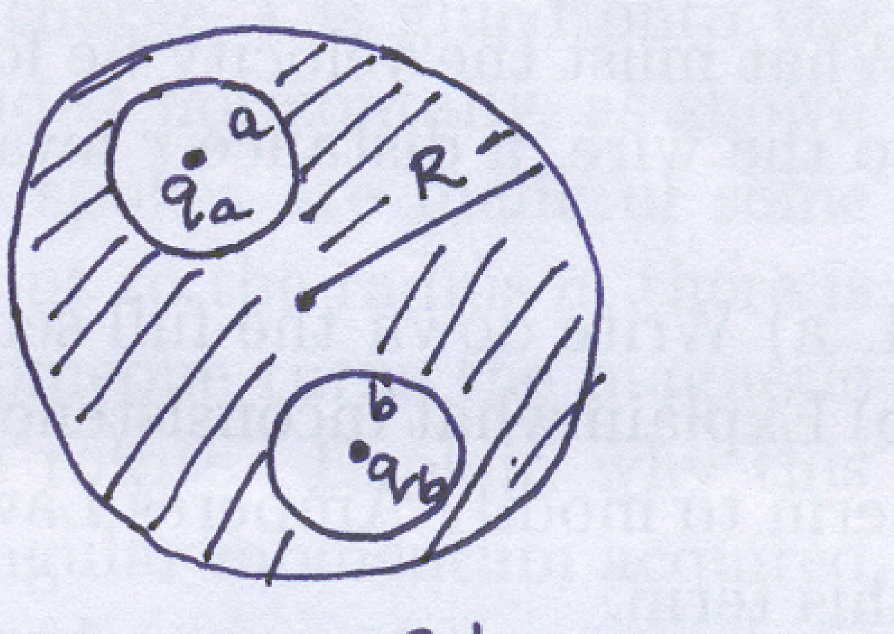


Fig 2b

2. (a) A conductor at potential $V = 0$ has the shape of an infinite plane except for a hemispherical bulge of radius a. A charge q is placed at a distance p from the plane (or a distance $p - a$ from the top of the bulge)(Fig 2a). What is the force on the charge ?
(b) Two spherical cavities , of radius a and b are hollowed out from the interior of a (neutral) conducting sphere of radius R. At the center of each cavity a point charge is placed, call these charges q_a and q_b. (Fig 2b)
(i) Find the surface charges σ_a, σ_b and σ_R.
(ii) What is the electric field outside the conductor?
(iii) What is the field within each cavity?

(iv) What is the force on q_a and q_b ?
(v) Which of these answers would change if a third charge q_c were brought near the conductor ?

3 (a) Show that if a particle of charge q and mass m moves in a time independent electric field $\mathbf{E} = -\nabla\phi(x, y, z)$ and *any* magnetic field, then the energy $\frac{1}{2}mv^2 + q\phi$ is a constant, where v is the magnitude of the velocity of the particle.
(b) Suppose the particle moves along the x-axis in the electric field $\mathbf{E} = Ae^{-\frac{t}{\tau}}\hat{\mathbf{x}}$ where A and τ are both constants. Suppose that the magnetic field is zero along the x axis and $x(0) = \dot{x}(0) = 0$. Find $x(t)$.
(c) In (b) is $\frac{1}{2}mv^2 - qxAe^{-\frac{t}{\tau}}$ a constant ? Indicate your reasoning briefly.
(d) A particle with charge q is travelling with velocity $\mathbf{v}$ parallel to a wire with uniform linear charge distribution λ per unit length. The wire also carries a uniform current I in the same direction as the velocity of the particle. What must the velocity be for the particle to travel in a straight line parallel to the wire, a distance r away?

4. a) Write down the full set of Maxwell's equations in differential form.
b) Explain what inconsistency led Maxwell to add the "displacement current" term to modify Amperé's Law and how consistency is restored on addition of this term.
c) Derive the continuity equation : $\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$ from Maxwell's equations, where the symbols have their usual meanings. Show that the above equation represents the conservation of charge.
d) Show that a magnetic field $\mathbf{B}$ and electric field $\mathbf{E}$ that is a solution to Maxwell's equations can always be written as

$$\mathbf{B} = \nabla \times \mathbf{A}$$

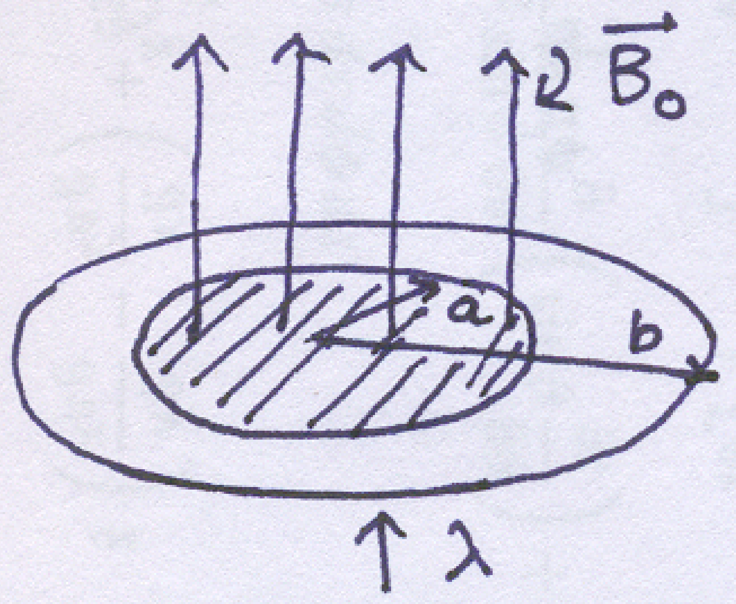
$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$$

where ϕ is a scalar function and $\mathbf{A}$ is a vector field.
(e) Show that , for Maxwell's equations in vacuum, each Cartesian component of $\mathbf{E}$ and $\mathbf{B}$ satisfies the 3-D wave equation

$$\nabla^2 f = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

with $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

5. A very small circular loop of radius a is initially coplanar and concentric with a much larger loop of radius b ($a \ll b$). A constant current I is passed in the large loop, which is kept fixed in space, and the small loop is rotated with angular velocity ω about a diameter. The resistance of the small loop is R and its self-inductance is negligible.
(a) Calculate the current in the small loop as a function of time.
(b) Calculate how much torque must be exerted on the small loop in order to rotate it.
(c) Calculate the induced emf in the large loop as a function of time.



6. (a) Consider the following situation. A line charge λ is glued onto the rim of a wheel of radius b , which is then suspended horizontally, as shown in the figure, so that it is free to rotate. (The spokes are made of some non-conducting material). In the central region, out to the radius a, there is a uniform magnetic field $\mathbf{B_0}$ pointing up. Now someone turns the magnetic field off. It is observed that the wheel starts to rotate. Explain why this happens, find the direction of rotation and the angular momentum acquired by the wheel. It appears that the system acquired an angular momentum without the application of an external torque. Explain whether this is consistent with the conservation of angular momentum.
(b) Write down the real electric and magnetic fields for a monochromatic plane wave of amplitude E_0 , frequency ω and phase angle zero that is travelling in the negative x direction and polarized in the z-direction. Find the time average (over a cycle) of the energy density and the Poynting vector for such a wave. What does the Poynting vector represent physically ?