Indian Statistical Institute, Bangalore
B. Math (Hons.) Second Year
First Semester - Ordinary Differential Equations

Mid-Semester Exam — Date: 21st February 2024
Maximum marks: 30 — Duration: 2 hours

Answer any six, each question carries 5 marks

1. Solve $y' + Py = Qy^n$ for any $n \in \mathbb{N} \cup \{0\}$ and use it to solve $xy' + y = x^4y^3$.
2. Find continuous functions P and Q so that e^x and xe^x are solutions of $y'' + Py' + Qy = 0$. Are P and Q unique for this property. Justify your answer.
3. Let p and q be constants. Reduce $x^2y'' + xpy' + qy = 0$ to a linear equation with constant coefficients and use it to solve $x^2y'' + 2xy' - 12y = 0$.
4. Considering power series method for the first order equation $(1 + x)y' = py$, prove $(1 + x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \cdots + \frac{p(p-1)\cdots(p-n)}{n!}x^n + \cdots$ for $|x| < 1$.
5. Let y be a solution of $y'' + Py' + Qy = 0$ on $[a, b]$ where P and Q are continuous functions on $[a, b]$. If y' and y'' are also solutions of $y'' + Py' + Qy = 0$ on $[a, b]$, then determine y.
6. Prove $x^2y''+xy'+(x^2-1/4)y = 0$ has two independent Frobenius series solutions using Frobenious method.
7. Prove that J_{p+1} has a zero between two positive zeros of J_p. Can J_p and J_{p+1} have a common zero. Justify your answer.
8. Let y_1, y_2 be solutions of second order homogeneous equation $y''+Py'+Qy = 0$. Let (y_1, z_1) and (y_2, z_2) be solutions of the corresponding system of first order equations $y' = z; z' = -Pz-Qy$. Prove that Wronskian of y_1, y_2 and Wronskian of (y_1, z_1) and (y_2, z_2) are same.