

Indian Statistical Institute, Bangalore

B. Math (Hons.) Second Year

Second Semester - Ordinary Differential Equations

Back Paper Exam

Date: 05th June 2024

Total marks: 50

Duration: 3 hours

Each question carries 5 marks

1. Find a condition for the equation $Mdx + Ndy = 0$ to have an integrating factor that is a function of $z = xy$.
2. Prove that Wronskian of solutions of any second order homogeneous linear differential equation is either zero or nowhere zero.
3. Solve $y'' + y = 2 \cos x$.
4. Find all Frobenius series solutions of $x^2y'' + xy' + (x^2 - 1/4)y = 0$.
5. Solve $x' = 4x - 2y$ and $y' = 5x + 2y$.
6. Prove $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.
7. State and prove Sturm comparison Theorem.
8. Find $y(0.5)$ by Euler's method for $y' = 2x + 2y$ with $h = 0.1$ and $y(0) = 1$.
9. Prove that the existence of a Liapunov function is sufficient for the stability of the critical point.
10. Prove that the system $x' = -y + x(1 - x^2 - y^2)$, $y' = x + y(1 - x^2 - y^2)$ has a closed path C and all other paths approach C .