BACK PAPER: NUMERICAL COMPUTING

(1) (6+6+6=18 points) Write down the output of the following commands in octave.
   (a) x=[5 3 2]; A=diag(2*ones(1,4))+diag(ones(1,3),1)+diag(x,-1); disp(A(1:3,2:4));
   (b) A=[2 1;4 3];B=[1 0;0 1]+diag([4 5]); disp(A.*B); disp(A*B);
   (c) c=[1 2 3 4]; r=polyval(c,-1); disp(r);

(2) (5+6+7=18 points) Describe what the following commands in octave do:
   (a) linspace
   (b) chol
   (c) ode45

(3) (4+4+10=18 points) Write down a command or a short code to achieve the following goals:
   (a) Display the plot of the function $f(x) = sin(x) + e^x$ for x between $-\pi$ and π.
   (b) Create a $n \times 2$ random matrix.
   (c) Given a $n \times 2$ matrix A, write a short code which will give a polynomial which interpolates the n points whose co-ordinates are given by the rows of A.

(4) (10 points) The function $y = \frac{x}{c_1 x + c_2 e^x}$ can be transformed into a linear relationship $z = c_1' w + c_2'$ with the change of variable $z = \frac{1}{y}$ and $w = \frac{e^x}{x}$. Write an "xlinxFit" function that calls linefit to fit data to $y = \frac{x}{c_1 x + c_2 e^x}$.

(5) (18 points) Write a function betatrap that uses the Trapezoid rule to evaluate
$$\beta(m; n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$$
using N (which is also an input variable) panels.

(6) (18 points) Write down an octave function to find a solution to the differential equation
$$y' = e^{y-t} + y, \; y(0) = 0$$
at t=2 using the stepsize h (which is a input variable for the function) following Runge-Kutta method.