GROUP THEORY (BMath-II, Back paper)

Answer as many questions as you like, for a maximum score of 50. Total time 3 *hours.*

You may use any result covered in the class without proving it. Use only results and notions covered in the course.

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1. Let p, q be primes, not necessarily distinct. Classify all groups of order pq. (10)

2. Let D_n be the dihedral group of order $2n$. Find the center of D_n. (10)

3. Let Q denote the group of quaternions. Prove that there is no injective homomorphism $Q \to S_4$. (8)

4. Is D_{12} isomorphic to $S_3 \times V_4$? Explain. Here V_4 denotes the Klein four group.(8)

5. Let G be a group of order 57. Assume G is not cyclic. How many elements in G have order 3? Explain. (8)

6. Determine if the dihedral group D_{109} of order 218 is nilpotent. (10)

7. Let G be a group of order 231. Prove that G gas a unique subgroup of order 11 and that this subgroup is contained in $Z(G)$, the center of G. (3+7)

8. Let $[n] := \{1, 2, \cdots, n\}$. Then $[m] \subset [n]$ for $m \leq n$. Consider the action of S_n on the elements of $[n]$. For $m \leq n$ let $S(m) := \{\sigma \in S_n | \sigma([m]) = [m]\}$. Prove that $S(m)$ is a subgroup of S_n. Determine this subgroup explicitly. (3+7)

9. Let $p > 3$ be a prime and G be a group of order $4p$. Prove that G is not simple. (8)

10. Let G be a finite non-abelian group having the property that there is a subgoup of order d for every divisor d of $|G|$. Prove that G is not simple. (10)