GEOMETRY MIDTERM EXAM

Attempt all questions. Marks will be deducted if you use a theorem to solve a problem but do not quote it (correctly).

(1) Let $\mathcal{H}$ be a hyperbola with major axis PQ, whose midpoint is O. Let the perpendicular at P to the major axis meet an asymptote at a point R. Prove that the circle with centre O and radius OR cuts the major axis of $\mathcal{H}$ at its foci. (10 marks)

(2) Consider three disjoint circles of unequal radii, whose centres are non collinear, in the plane. Draw pairs of tangents to each pair of circles and assume that the point of intersection of each pair of tangents lies beyond the two circles. Prove that the three intersection points are collinear. (10 marks)

(3) State and prove the dual of the Theorem of Pappus. (10 marks)

(4) Let $\mathcal{C}$ be a nondegenerate projective conic through the vertices of a quadrilateral $PQRS$. It is given that the tangents to $\mathcal{C}$ at P and R meet at a point X on the line QS. Show that the tangents to $\mathcal{C}$ at Q and S meet at point Y on PR. (10 marks)

(5) Let PQ be the diameter of a circle $\mathcal{C}$. Consider two chords PR and QS of $\mathcal{C}$ which intersect (after extending, if needed) at T. Prove that the circle RST is orthogonal to the circle $\mathcal{C}$. (10 marks)

(6) Let $\mathcal{F}$ be an Apollonian family of circles defined by the points P and Q. Let R be any point on the radical axis of $\mathcal{F}$. Show that the tangents from R to all circles in $\mathcal{F}$ are of equal length. (10 marks)