Differential Geometry

B. Math. III

Back-Paper Examination

Instructions: All questions carry equal marks.

1. Let $\gamma(t)$ be a regular curve whose derivatives with respect to the parameter t will be denoted by 'dots'. Define its curvature κ. Prove that
$$\kappa = \frac{||\ddot{\gamma} \times \dot{\gamma}||}{||\dot{\gamma}||^3}.$$

2. Let γ and $\tilde{\gamma}$ be two unit-speed curves in $\mathbb{R}^3$ with the same strictly positive *signed curvature* κ_s and the same torsion $\tau(s)$. Prove that there is a rigid motion M of $\mathbb{R}^3$ such that $\gamma(s) = M(\tilde{\gamma}(s))$ for all s.

3. Let $S \subset \mathbb{R}^3$ be an oriented regular surface. Define the Gauss map of S and prove that its differential at each point $p \in S$ is a self-adjoint (with respect to the First Fundamental Form of S) linear operator on the tangent space $T_p(S)$ of S at p.

4. Define Gaussian curvature of an oriented surface. If S is a compact oriented surface, then prove that there exists a point P such that the Gaussian curvature of S at P is strictly positive.

5. Let $S^2 \subset \mathbb{R}^3$ denote the unit sphere with origin as its centre. Let P be a plane passing through origin and let C be the curve given by the intersection of S^2 and P. Parametrize C and find its normal and geodesic curvature at each point.