Semestral Exam B.Math III Year (Differential Geometry) 2016

Attempt all questions. Each question carries 15 marks. Books and notes maybe consulted. Results proved in class, or propositions (with or without proof) from the class notes maybe used after quoting them. Results from exercises in the notes or Pressley's book, which haven't been solved in class must be proved in full if used.)

1. Consider the torus:
$$T^2 = \left\{((2+\cos s)\cos t,\ (2+\cos s)\sin t,\ \sin s) \in \mathbb{R}^3 :\ s,t \in [0,2\pi]\right\}$$
Compute the normal and geodesic curvature of the curve $c : [0,4\pi] \to T^2$ defined by:
$$c(t) = \left(2\cos\frac{t}{2}, 2\sin\frac{t}{2}, 1\right)$$
where t is the arc length parameter.

2. Consider the ellipsoid:
$$X = \left\{(x,y,z) \in \mathbb{R}^3 :\ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1\right\}$$
(i): Find a coordinate chart (ϕ, V) for X (where V is an open subset of $\mathbb{R}^2$) with the image $\phi(V)$ being the open subset of X given by $U := X \setminus \{(x,0,z) : x \geq 0\}$. (7 marks)

(ii): Compute the second fundamental form for X on this chart. (8 marks)

3. Let $X \subset \mathbb{R}^3$ be a connected minimal surface (i.e. a surface of identically vanishing mean curvature). Show that X is a subset of a plane iff its scalar curvature $K(x)$ vanishes identically.

4. Let X be a compact connected surface of genus $g \geq 2$. Let the minimum of the scalar curvature function K on X be denoted by K_{min}. Prove that $K_{\text{min}} = -\mu$ where:
$$\mu \geq \frac{4\pi(g-1)}{A}$$
and A is the area of X.