Complex Analysis Mid Term

September 20 2010

There are 5 questions of 6 points each. Please look over the entire paper before attempting to answer as some questions may be easier than others.

1. Prove that an entire function with a positive real part is constant.

2. Compute the residues of the following functions at each of their isolated singularities –

$$(a) \frac{cos(z)}{1+z+z^2} \qquad (b) \frac{z^5}{(z^2-1)^2}$$

3. Let γ denote the unit circle traversed counter clockwise. Evaluate

$$(a) \oint_\gamma \frac{e^{-z}}{(z-1/2)^2} \qquad (b) \oint_\gamma \frac{cos(z^3)}{(z-3)}$$

4. Let f be a holomorphic map of the open unit disc to itself. Prove that

$$|f'(z)| \leq \frac{1}{1-|z|}$$

for all z in the unit disc.

5. Give an example of a complex function which satisfies the Cauchy-Riemann equations but is not holomorphic.