BACKPAPER EXAMINATION
B. MATH III YEAR, I SEMESTER 2012-2013
COMPLEX ANALYSIS

The 7 questions below carry a total of 110 marks. The maximum you can score is 100. Time limit is 3 hours.

Notations: U is the open unit disk $\{z : |z| < 1\}$. $H(\Omega)$ is the set of all holomorphic functions on Ω.

1. a) Let R_1 and R_2 be the radii of convergence of $\sum a_n z^n$ and $\sum b_n z^n$ respectively. Assume that $0 < R_1 < \infty$ and $0 < R_2 < \infty$. Find the radius of convergence of $\sum a_n b_n z^n$.

b) Find the radius of convergence of $\sum e^{e^{(n^2+in^3\pi/3)}} z^n$. [5+10]

2. Does there exist a one-to-one holomorphic function f on U such that f has a zero of order 3 at 0? Give full statement of any theorem you want to use. [10]

3. Let $f_n \in H(B(0, 1 + \delta))$ for some $\delta > 0$ $\forall n \geq 1$. Assume that $f_n(0) = 0$ $\forall n$. If $f_n(z) \to z$ uniformly for $|z| \leq 1$ show that there is a positive integer k such that $f_n(z) \neq 0$ $\forall n \geq k$ $\forall z \in U \backslash \{0\}$. [15]

4. Show that $u(x, y) = \sinh(x) \cos(y)$ is harmonic on $\mathbb{C}$ and find a harmonic conjugate. [20]

5. Evaluate $\int_{-\infty}^{\infty} \frac{1-x^2}{1+6x^4} dx$ using contour integration. [20]

6. Prove that $Log(z) - Log(z - i) - Log(\frac{z}{z-i}) = 0$ for $\text{Re}(z) > 0$. [20]

Hint: $e^{L.H.S.} = 1$.

7. Find the number of zeros of the poynomial $1 - 2z^2 + 10z^4 - 100z^7$ that lie in U. [10]