SEMESTRAL EXAMINATION
B. MATH III YEAR, I SEMESTER 2012-2013
COMPLEX ANALYSIS

The 7 questions below carry a total of 110 marks. The maximum you can score is 100. Time limit is 3 hours.

Notations: U is the open unit disk $\{z : |z| < 1\}$. $H(\Omega)$ is the set of all holomorphic functions on Ω.

1. Let $\gamma(t) = e^{2\pi it}, 0 \le t \le 1$. If z_1 and $z_2 \in U$ show that $\int\limits_{\gamma} \frac{1}{(\zeta - z_1)(\zeta - z_2)} d\zeta = 0$. [10]

2. If $f \in H(U)$ and f is bounded prove that $\sum\limits_{n=0}^{\infty} \left| \frac{f^{(n)}(0)}{n!} \right|^2 < \infty$. If $f \in H(U)$ and $\sum\limits_{n=0}^{\infty} \left| \frac{f^{(n)}(0)}{n!} \right|^2 < \infty$ can we conclude that f is bounded? [20]

Hint: Consider $\int\limits_{0}^{2\pi} \left| f(re^{it}) \right|^2 dt$.

3. Let $p(z) = \sum\limits_{j=0}^{N} c_j z^j$ with $c_N \neq 0$. If $R = \max\{1, \frac{1}{|c_N|} \sum\limits_{j=0}^{N-1} |c_j|\}$ show that all the zeros of p lie on $B(0, R)$. [15]

4. Let $f : U \to U$ be holomorphic. If f has two or more fixed points show that f is the identity map. [20]

Hint: first consider the case when one of the fixed points is 0.

5. Prove that $\int\limits_{-\infty}^{\infty} e^{itx} e^{-x^2/2} dx = \sqrt{2\pi} e^{-t^2/2}$ for every real number t. [20]

6. Let $f \in H(U \backslash \{0\})$ and $f(\frac{1}{n}) = 0$ for all positive integers n. Show that either f is identically 0 or its range is dense in $\mathbb{C}$. [10]

7. Ler $R = \frac{p}{q}$ be a rational function such that $\deg(q) > \deg(p) + 1$. Show that the sum of all the residues of R is 0. [15]